

Signals and Systems Homework 4

March 19, 2016

1 Problem 1

Determine the Laplace transform of each of the following signals and specify the corresponding regions of convergence.

(1) $f(t) = e^{-a|t|}$

(2) $f(t) = |t|e^{-a|t|}$

(3) $f(t) = \sin(w|t|)$

(4) $f(t) = \cos(w|t|)$

(5) $f(t) = e^{-a|t|} \sin(w|t|)$

(6) $f(t) = e^{-at} \cos(w|t|)$

(7) $f(t) = |t| \sin(w|t|)$

(8) $f(t) = |t|^n e^{-a|t|}$, n is positive integer.

2 Problem 2

Determine the Laplace transform of each of the following signals and specify the corresponding regions of convergence, then plot the poles and zeros.

(1) $x(t) = e^{-4t}u(t) + e^{-5t} \sin(5t)u(t)$

(2) $x(t) = e^{2t}u(-t) + e^{3t}u(-t)$

(3)

$$x(t) = \begin{cases} t & 0 \leq t \leq 1 \\ 2-t & 1 \leq t \leq 2 \end{cases}$$

3 Problem 3

A system is described by the following differential equation (see below). Find the expression for the transfer function of the system, $Y(s)/X(s)$, assuming zero initial conditions.

$$\frac{d^3y}{dt^3} + 3\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + y = \frac{d^3x}{dt^3} + 4\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 8x$$

Consider what if the initial conditions are not zero.

4 problem 4

Let $x(t)$ be the sampled signal specified as

$$x(t) = \sum_{n=0}^{\infty} e^{-nT} \delta(t - nT)$$

where $T > 0$

- (a) Determine $X(s)$, including its region of convergence.
- (b) Sketch the pole-zero plot for $X(s)$
- (c) Use geometric interpretation of the pole-zero to argue that $X(j\Omega)$ is periodic.