Signals and Systems Homework 6

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1 Problem 1

Steady state of LTI systems

The transfer function of an LTI system is

$$H(s) = \frac{Y(s)}{X(s)} = \frac{s-1}{s^2+3s+2}$$

If the input to this system is $x(t) = 1 + \cos(t + \pi/4), -\infty < t < \infty$, what is the output $y(t)$ in the steady state.

2 Problem 2

Fourier series of sampling delta

The periodic signal

$$\delta_{T_s}(t) = \sum_{m=-\infty}^{\infty} \delta(t - mT_s - \Delta)$$

will be very useful in the sampling of continuous-time signal.

(a) Find the Fourier series of the signal-that is

$$\delta_{T_s}(t) = \sum_{k=-\infty}^{\infty} \Delta_k e^{jk\Omega_s t}$$

find the Fourier coefficients Δ_k.

(b) Plot the magnitude line spectrum of this signal.

(c) Plot $\delta_{T_s}(t)$ and its corresponding line spectrum Δ_k as functions of time and frequency. Are they both periodic? How are their periods related? Explain.

3 Problem 3

Manipulation of periodic signals

Let the following be the Fourier series of a periodic signal $x(t)$ of period T_0 (fundamental frequency $\Omega_0 = 2\pi/T_0$):

$$x(t) = \sum_{k=-\infty}^{\infty} X_k e^{j\Omega_0 kt}$$

Consider the following functions of $x(t)$, and determine if they are periodic and what are their periods if so:

(1) $y(t) = 2x(t) + 3$
(2) $z(t) = x(t-2) + x(t)$
(3) $w(t) = x(2t-1)$

Express the Fourier series coefficients Y_k, Z_k and W_k in terms of X_k.

4 Problem 4

Windowing and music sounds-MATLAB

In the computer generation of musical sounds,pure tones need to be windowed to make them more interesting.Windowing mimics the way a musician would approach the generation of a certain sound.Increasing the richness of the harmonic frequencies is the result of the windowing,as we will see in this problem.Consider the generation of a musical note with frequencies around $f_A = 880Hz$.Assume our "musician" while playing this note uses three strokes corresponding to a window $w_1(t) = r(t) - r(t-T_1) - r(t-T_2) + r(t-T_0)$,so that the resulting sound would be the multiplication,or windowing,of a pure sinusoid $\cos(2\pi f_A t)$ by a periodic signal $w(t)$,with $w_1(t)$ a period that repeats every $T_0 = 5T$ where T is the period of the sinusoid.Let $T_1 = T_0/4$ and $T_2 = 2T_0/4$.

(a) Analytically determine the Fourier series of the window $w(t)$ and plot its line spectrum using MATLAB.Indicate how you would choose the number of harmonics needed to obtain a good approximation to $w(t)$

(b) Use the modulation or the convolution properties of the Fourier series to obtain the coefficients of the product $s(t) = \cos(2\pi f_A t)w(t)$.Use Matlab to plot the line spectrum of this periodic signal and again determine how many harmonic frequencies you would need to obtain a good approximation to $s(t)$.

(c) The line spectrum of the pure tone $p(t) = \cos(2\pi f_A t)$ only displays one harmonic,the one corresponding to the $f_A = 880Hz$ frequency.How many more harmonics does $s(t)$ have ?To listen to the richness in harmonics use the MATLAB function sound to play the sinusoid $p(t)$ and $s(t)$(use $F_s = 2 \times 880Hz$ to play both).

(d) Consider a combination of notes in a certain scale; for instance, let

$$p(t) = \sin(2\pi \times 440t) + \sin(2\pi \times 550t) + \sin(2\pi \times 660t)$$

Use the same windowing $w(t)$,and let $s(t) = p(t)w(t)$.Use MATLAB to plot $p(t)$ and $s(t)$ and to compute and plot their corresponding line spectra.Use sound to play $p(nT_s)$ and $s(nT_s)$ using $F_s = 1000$.

5 Problem 5

Computation of π-MATLAB

As you know,π is an irrational number that can only be approximated by a number with a finite number of decimals.How to compute this value recursively is a problem of theoretical interest.In this problem we show that the Fourier series can provide that formulation.

(a) Consider a train of rectangular pulses $x(t)$,with a period

$$x_1(t) = 2[u(t+0.25) - u(t-0.25)] - 1 \quad -0.5 \leq t \leq 0.5$$

and period $T_0 = 1$.Plot the periodic signal and nd its trigonometric Fourier series.
(b) Use the above Fourier series to nd an innite sum for π.
(c) If π_N is an approximation of the innite sum with N coefficients,and π is the value given by MATLAB,find the value of N so that π_N is 95% of the value of π given by MATLAB.