<u>Pr 9.7</u> (a) If $T_s = 0.1$ the discrete-time signal is

$$x(0.1n) = [u(t) - u(t-1)] \mid_{t=0.1n} = \begin{cases} 1 & 0 \leq 0.1n \leq 1 \quad \text{or} \quad 0 \leq n \leq 10 \\ 0 & \text{otherwise} \end{cases}$$

(b) Expressing $x[n]$ as indicated, then $N = 11$.

(c) The Laplace transform of the sampled signal is

$$\begin{aligned} X_s(s) &= \sum_{n=0}^{10} \mathcal{L}[\delta(t - nT_s) \\ &= \sum_{n=0}^{10} e^{-0.1ns} \\ &= \frac{1 - e^{-1.1s}}{1 - e^{-0.1s}} \end{aligned}$$

(d) The z-transform of the discrete-time signal is

$$X(z) = \sum_{n=0}^{10} z^{-n} = \frac{1 - z^{-11}}{1 - z^{-1}}$$

(e) To transform $X_s(s)$ into $X(z)$ we let $z = e^{0.1s}$.

Pr 9.10 (a) The given signal can also be written

$$x[n] = \begin{cases} 1 & n \geq 0 \text{ and even} \\ 0 & \text{otherwise} \end{cases}$$

(b) Using the above expression for $x[n]$, we have

$$\begin{aligned} X(z) &= \sum_{n=0,\text{ even}}^{\infty} 1\, z^{-n} \\ &= \sum_{m=0}^{\infty} 1\, z^{-2m} = \frac{1}{1-z^{-2}} \qquad |z| > 1 \end{aligned}$$

where we let $n = 2m$ to find the final expression.
(c) The z-transform of $x[n]$ is also obtained by using its linearity

$$\begin{aligned} X(z) &= 0.5\mathcal{Z}[u[n]] + 0.5\mathcal{Z}[(-1)^n u[n]] \\ &= \frac{1}{2(1-z^{-1})} + 0.5\sum_{n=0}^{\infty}(-z^{-1})^n \\ &= \frac{1}{2(1-z^{-1})} + \frac{1}{2(1+z^{-1})} \\ &= \frac{1}{1-z^{-2}} \qquad |z^{-1}| < 1 \text{ or } |z| > 1 \end{aligned}$$

(c) To find the poles and zeros let

$$X(z) = \frac{z^2}{z^2-1}$$

with poles $z = \pm 1$, and zeros $z = 0$, double.

Pr 9.11 (a) The Z-transform of the difference equation

$$y[n] = x[n] - 0.5y[n-1] \qquad n \geq 0$$

with initial condition $y[-1]$ is

$$Y(z) = X(z) - 0.5(z^{-1}Y(z) + y[-1])$$

so that

$$Y(z) = \frac{X(z)}{1 + 0.5z^{-1}} - \frac{0.5y[-1]}{1 + 0.5z^{-1}}$$

(b) If $X(z) = 1$, $y[-1] = 2$ then $Y(z) = 0$ and therefore $y[n] = 0$ for $n \geq 0$ but $y[-1] = 2$.
If $X(z) = 1$ or $x[n] = \delta[n]$ and $y[-1] = 2$ the difference equation is

$$y[n] = \delta[n] - 0.5y[n-1] \qquad n \geq 0$$

and can be solved recursively

$$\begin{aligned} y[0] &= 1 - 0.5 \times 2 = 0 \\ y[1] &= 0 - 0.5 \times 0 = 0 \\ y[2] &= 0 - 0.5 \times 0 = 0 \\ &\vdots \end{aligned}$$

(c) If $y[-1] = 0$ and $x[n] = \delta[n]$ then we can compute $y[n] = h[n]$, i.e., the impulse response. The corresponding transfer function is then from the equation for $Y(z)$:

$$H(z) = \mathcal{Z}[h[n]] = \frac{Y(z)}{X(z)} = \frac{1}{1 + 0.5z^{-1}}$$

If we want $y[n] = \delta[n] + 0.5\delta[n-1]$ or $Y(z) = 1 + 0.5z^{-1}$ then

$$X(z) = \frac{Y(z)}{H(z)} = (1 + 0.5z^{-1})^2 = 1 + z^{-1} + 0.25z^{-2}$$

which gives $x[n] = \delta[n] + \delta[n-1] + 0.25\delta(n-2)$

<u>Pr 9.17</u> (a) The signal $x[n] = \delta[n] + \delta[n-1] + \delta[n-2]$ has a Z-transform

$$X(z) = 1 + z^{-1} + z^{-2}$$

(b) Then

$$Y(z) = X^2(z) = (1 + z^{-1} + z^{-2})^2 = 1 + 2z^{-1} + 3z^{-2} + 2z^{-3} + z^{-4}$$

The convolution of the coefficients of $X(z)$, or $x[n]$, with themselves gives the sequence

$$y[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2] + 2\delta[n-3] + \delta[n-4]$$

The length of $y[n]$ is twice that of $x[n]$ minus one, or $2\times3-1 = 5$ so that $Y(z)$ is a fourth -degree polynomial. The above result is verified using MATLAB

```
%% Pr 9.17
x=[1 1 1];
y=conv(x,x); N=length(y);
x1=[x zeros(1,N-3)]
n=0:N-1;
figure(1)
subplot(211)
stem(n,x1),ylabel('x[n]');grid
subplot(212)
stem(n,y);ylabel('y[n]'); grid
```

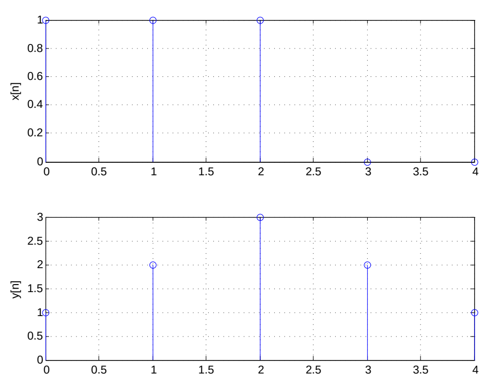

Figure 9.4: .

<u>Pr 9.18</u> Writing $X(z)$ using terms found in tables, its partial fraction expansion is

$$\begin{aligned} X(z) &= \frac{2-z^{-1}}{2(1+0.25z^{-1})(1+0.5z^{-1})} \\ &= \frac{A}{1+0.25z^{-1}} + \frac{B}{1+0.5z^{-1}} \end{aligned}$$

corresponding to the poles at -0.25 and -0.5. The coefficients of the expansion are

$$\begin{aligned} A &= \frac{2-z^{-1}}{2(1+0.5z^{-1})}\,|_{z^{-1}=-4} = -3 \\ B &= \frac{2-z^{-1}}{2(1+0.25z^{-1})}\,|_{z^{-1}=-2} = 4 \end{aligned}$$

so that

$$X(z) = \frac{-3}{1+0.25z^{-1}} + \frac{4}{1+0.5z^{-1}}$$

and the inverse is

$$x[n] = [-3(-0.25)^n + 4(-0.5)^n]u[n]$$

and in the steady–state it is zero.

<u>Pr 9.19</u> (a) $F(z)$ is a proper rational function in positive powers of z as its numerator is of lower order than the denominator. If we convert it into negative powers, z^{-1}, we have

$$F(z) = \frac{z^{-2}(1 + z^{-1})}{1 - z^{-1}}$$

which is not proper rational in z^{-1} as its numerator is of higher order than its denominator.
(b) Using the above expression we have that

$$F(z) = \frac{z^{-2}}{1 - z^{-1}} + \frac{z^{-3}}{1 - z^{-1}}$$

which gives

$$f[n] = u[n-2] + u[n-3]$$

given that $1/(1 - z^{-1})$ is the Z-transform of $u[n]$ and z^{-2} and z^{-3} delay $u[n]$ by 2 and 3 samples.