Solution 3

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Problem 1:
(a) Linear(not memoryless,not time-invariant,not causal,not stable)
(b) Linear,causal,stable(not memoryless,not time-invariant)
(c) Linear,stable(not memoryless,not time-invariant,not causal)
(d) Time invariant,Linear,causal(not memoryless,not stable)
Problem 2:
(a) From the given information,we see that $h(t)$ is nonzero only for $0 \leq t \leq \infty$.Therefore,

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau \tag{1}$$

$$= \int_{-\infty}^{\infty} e^{3t}(u(t-\tau-3) - u(t-\tau-5))d\tau \tag{2}$$

we can show that $(u(t-\tau-3) - u(t-\tau-5))$ is nonzero in the range $(t-5) < \tau < (t-3)$.Therefore,for $t \leq 3$,the above integral is zero.For $3 < t \leq 5$,the above integral is

$$y(t) = \int_{0}^{t-3} e^{3t}dt = \frac{e^{3t-9}-1}{3} \tag{3}$$

For $t > 5$,the integral is

$$y(t) = \int_{t-5}^{t-3} e^{3t}dt = \frac{e^{3t-9}-e^{3t-15}}{3} \tag{4}$$

(b) By differentiating $x(t)$ with respect to time we get

$$\frac{dx(t)}{dt} = \delta(t-3) - \delta(t-5) \tag{5}$$

$$g(t) = \frac{dx(t)}{dt} * h(t) = e^{3(t-3)}u(t-3) - e^{3(t-5)}u(t-5) \tag{6}$$

(c) $g(t) = \frac{dy(t)}{dt}$
Problem 3:

(a)Note that

$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau - 2) d\tau \tag{7}$$

$$= \int_{-\infty}^{t-2} e^{-(t-2-\tau')} x(\tau') d\tau' \tag{8}$$

Therefore,

$$h(t) = e^{-(t-2)} u(t-2) \tag{9}$$

(b) We have

$$y(t) = \int_{-\infty}^{\infty} h(\tau) x(t-\tau) d\tau \tag{10}$$

$$= \int_{2}^{\infty} e^{-(\tau-2)[u(t-\tau+1)-u(t-\tau-2)]} d\tau \tag{11}$$

then,we can get

$$y(t) = \begin{cases} 0, & t \leq 1 \\ 1 - e^{-(t-1)}, & 1 < t \leq 4 \\ e^{-(t-4)}(1 - e^{-3}), & t > 4 \end{cases} \tag{12}$$

Problem 4:
Note that

$$\frac{dx(t)}{dt} = -6e^{-3t} u(t-1) + 2e^{-3} \delta(t-1) \tag{13}$$

$$= -3x(t) + 2e^{-3} \delta(t-1). \tag{14}$$

Given that

$$x(t) = 2e^{-3t} u(t-1) \to y(t) \tag{15}$$

we know that $\frac{dx(t)}{dt} = -3x(t) + 2e^{-3}\delta(t-1)$ must yield $-3y(t) + 2e^{-3}h(t-1)$ at the ouput .Form the given information,we may conclude that $2e^{-3}h(t-1) = e^{-2t}u(t)$.Therefore,

$$h(t) = \frac{1}{2} e^{3} e^{-2(t+1)} u(t+1) \tag{16}$$