Solutions of Mid-term

April 18, 2016

1 Solution 1

From the frequency convolution property, we obtain

$$f^2(t) \Longleftrightarrow \frac{1}{2\pi} F(\omega) * F(\omega) \tag{1}$$

Because of the width property of the convolution, the width of $F(\omega) * F(\omega)$ is twice the width of $F(\omega)$. Repeated application of this argument shows that the bandwidth of $f^n(t)$ is nB Hz (n times the bandwidth of $f(t)$).

2 Solution 2

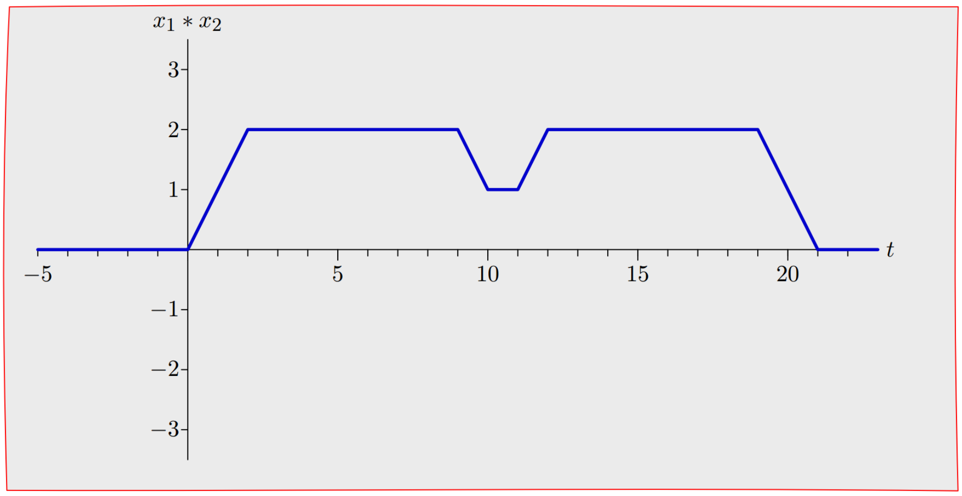

Figure 1: $x_1 * x_2$

3 Solution 3

$$x(t) = \sum_{n=0}^{\infty} (\frac{1}{2})^n (u(t-n) - u(t-n-\frac{1}{2}))$$

Apply Laplace transform, we have,

$$
\begin{aligned}
X(s) &= \int_{-\infty}^{\infty} \sum_{n=0}^{\infty} (\frac{1}{2})^n (u(t-n) - u(t-n-\frac{1}{2})) e^{-st} dt \\
&= \sum_{n=0}^{\infty} (\frac{1}{2})^n \int_{-\infty}^{\infty} (u(t-n) - u(t-n-\frac{1}{2})) e^{-st} dt \\
&= \sum_{n=0}^{\infty} (\frac{1}{2})^n \frac{1}{s} e^{-sn} (1 - e^{-s/2}) \\
&= \frac{1}{s} (1 - e^{-s/2}) \sum_{n=0}^{\infty} (\frac{1}{2} e^{-s})^n \\
&= \frac{1}{s} (1 - e^{-s/2}) \frac{1 - (\frac{e^{-s}}{2})^{n+1}}{1 - \frac{1}{2} e^{-s}} \\
&= \frac{1}{s} (\frac{1 - e^{-s/2}}{1 - \frac{1}{2} e^{-s}})
\end{aligned}
$$

This transform converges if

$$|\frac{1}{2} e^{-s}| < 1$$

which gives,

$$
\begin{aligned}
|e^{-\text{Re}(s)}| &< 2 \\
\text{Re}(s) &> -\ln 2
\end{aligned}
$$

4 Solution 4

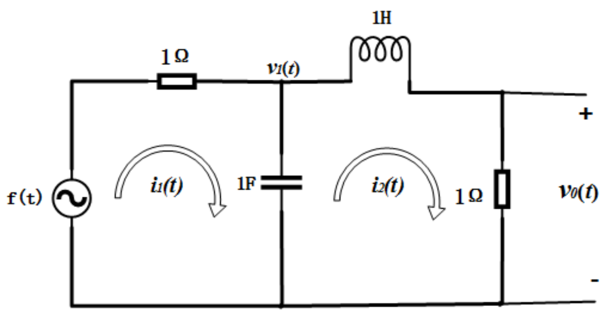

Figure 2: Pro. 4

At first,we have

$$
\begin{cases}
f(t) &= i_1(t) + v_1(t) \\
v_1(t) &= v_0(t) + v_0'(t) \\
v_0(t) &= i_1(t) - v_1'(t)
\end{cases}
$$

Apply the Laplace transform, we have

$$
\begin{cases}
F(s) &= I_1(s) + V_1(s) \\
V_1(s) &= V_0(s) + s.V_0(s) \\
V_0(s) &= I_1(s) - sV_1(s)
\end{cases}
$$

which gives,

$$
\begin{aligned}
&[1 + (1+s)^2] V_0(s) = F(s) \\
&\frac{V_0(s)}{F(s)} = \frac{1}{(s+1)^2 + 1}
\end{aligned}
$$

then we have the transfer function of system,

$$H(s) = \frac{1}{(s+1)^2+1}$$
$$h(t) = \sin(t)e^{-t}u(t)$$

and if $f(t) = te^{-t}u(t)$,then

$$F(s) = \frac{1}{(s+1)^2}$$
$$V_0(s) = F(s)H(s) = \frac{1}{(s+1)^2(s^2+2s+2)}$$

then the output is

$$v_0(t) = (te^{-t} - \quad e^{-t}\sin(t))u(t)$$

5 Solution 5

Take the Fourier transform of $h(t)$ to obtain the frequency response,

$$\begin{aligned} H(j\omega) &= e^{-j\omega T_1} + \epsilon e^{-j\omega T_2} \\ &= e^{-j\omega T_1}(1 + \epsilon e^{-j\omega(T_2-T_1)}) \end{aligned}$$

The magnitude function

$$\begin{aligned} |H(j\omega)| &= |e^{-j\omega T_1}||1 + \epsilon e^{-j\omega(T_2-T_2)}| \\ &= 1\sqrt{(1+\epsilon\cos\omega(T_2-T_1))^2 + \epsilon^2\sin^2\omega(T_2-T_1)} \\ &= \sqrt{1+2\epsilon\cos\omega(T_2-T_1)+\epsilon^2} \end{aligned}$$

oscillates between $1+\epsilon$ and $1-\epsilon$ with a period (in ω) of $2\pi/(T_2-T_2)$.From the magnitude plot, we can see that $\epsilon \approx 0.2$ and $2\pi/(T_2-T_1) \approx 1500/2$,so that $T_2 - T_1 \approx \frac{4\pi}{1500}$.
The angle function is

$$\angle H(j\omega) = -\omega T_1 + \angle(1 + \epsilon e^{-j\omega(T_2-T_1)})$$

Since ϵ is small compared to 1, the first term dominates the second, which oscillates about an average value near 0. Thus we can estimate T_1 from the average slope of the angle plot,

$$T_1 = \frac{\pi}{1500} \quad =0.0021$$

Then $T_2 \approx \frac{4\pi}{1500} + \frac{\pi}{1500} = \frac{\pi}{300}$. =0.0105