HW#3:

Problem 1:

We have introduced a number of general properties of systems. In particular, a system may or may not be
(1)Memoryless; (2)Time invariant; (3)Linear; (4)Causal; (5)Stable.

Determine which of these properties hold and which do not hold for each of the following continuous-time systems. Justify your answers in each example. y(t) denotes the system output and x(t) is the system input.

$(a)\ \ y(t) = \int_{-\infty}^{4t} x(\tau)\, d\tau$

$(b)\ y(t) = \begin{cases} 0 & t < 0 \\ x(t) + x(t-1) & t \geq 0 \end{cases}$

$(c)\ \ y(t) = x(\frac{t}{5})$

$(d)\ y(t) = \frac{dx(t)}{dt}$

Problem 2: Calculate the convolutions

2.1:

Let $x(t) = u(t-2) - u(t-6)$ and $h(t) = e^{3t}u(t)$

(a) Compute $y(t) = x(t) * h(t)$

(b) Compute $g(t) = \frac{dx(t)}{dt} * h(t)$

(c) How is $g(t)$ related to $y(t)$?

2.2:

Let $u(t)$ be the unit-step and

$$x(t) = (t+2)[u(t+2) - u(t)] + (-t+2)[u(t) - u(t-2)]$$

$$y(t) = (t+1)[u(t+1) - u(t)] + (-t+1)[u(t) - u(t-1)]$$

Compute h$(t) = x(t) * y(t)$.

Problem 3

Impulse response of a system:
Consider an system with input and output related through the equation:
($y(t)$ is the output and $x(t)$ is the input)

$$y(t) = \int_{-\infty}^{t} e^{-2(t-\tau)} x(\tau - 1) d\tau$$

What is the impulse response $h(t)$ for this system?

Problem 4

Impulse response of a LTI system:
Consider an LTI system S and a signal $x(t) = 2e^{-3t}u(t-2)$. If

$$x(t) \underset{S}{\rightarrow} y(t)$$

and

$$\frac{dx(t)}{dt} \underset{S}{\rightarrow} -3y(t) + e^{-2t}u(t)$$

What is the impulse response $h(t)$ for this system?