HW#6: Chapter 4

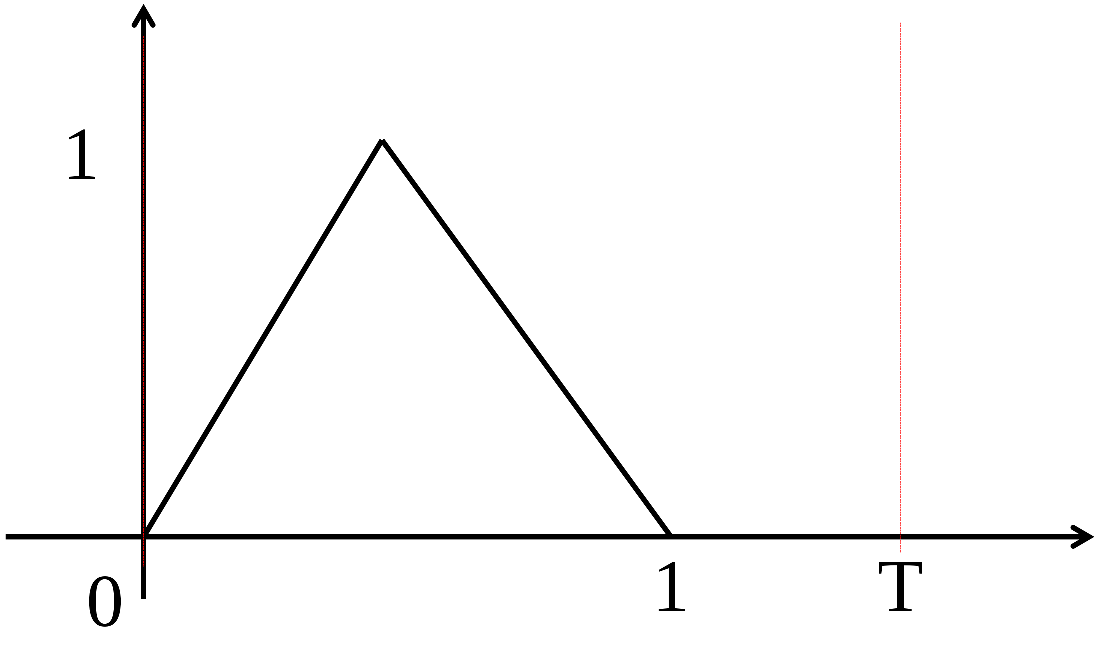
Prob. 1 :

(a) Given Fourier series $X_k = \frac{1}{T_0}\int_0^{T_0} x(t)e^{-jk\frac{2\pi}{T_0}}dt$, let $T_1 = MT_0$ and $\tilde{k} = Mk + r$, whre $r \in [0, M-1]$, we have another Fourier series:

$$Y_{\tilde{k}} = \frac{1}{T_1}\int_0^{T_1} x(t)e^{-j\tilde{k}\frac{2\pi}{T_1}}dt .$$

Please determine the relationship between the two set of Fourier series coefficients.

(b) Consider a periodic signal $x(t)$ of period $T \geq 1$, one period of which is shown below. Find the Fourier series and comment on your observations.



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Prob. 2 :

Steady state of an LTI system

The transfer function of an LTI system is $H(s) = \dfrac{Y(s)}{X(s)} = \dfrac{s-1}{s^2+3s+2}$

When the input to this system is $x(t) = 1 + \sin(t + \pi / 4), -\infty < t < \infty$, find the output $y(t)$ in the steady state.

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Prob. 3 :

Fourier series of sampling delta

The periodic signal

$$\delta_{T_s}(t) = \sum_{m=-\infty}^{\infty} \delta(t - mT_s - \Delta)$$

will be very useful in the sampling of continuous-time signal.

(a) Determine the Fourier series of the impulse train, i.e.

$$\delta_{T_s}(t) = \sum_{k=-\infty}^{\infty} \Delta_k e^{jk\Omega_s t}$$

(b) Plot the magnitude line spectrum of this signal.

(c) Plot $\delta_{T_s}(t)$ and its corresponding line spectrum Δ_k as functions of time and frequency. Are they both periodic? How are their periods related? Please explain.

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Prob. 4 :

Manipulation of periodic signals

Let the following be the Fourier series of a periodic signal $x(t)$ of period T_0 (fundamental frequency $\Omega_0 = 2\pi / T_0$):

$$x(t) = \sum_{k=-\infty}^{\infty} X_k e^{j\Omega_0 kt}$$

For the following signals, determine if they are periodic and what are their periods if so:

(1) $y(t) = 2x(t) - 3$

(2) $z(t) = x(t-2) + x(t)$

(3) $w(t) = x(2t-1)$

If periodic, express the Fourier series coefficients Y_k, Z_k and W_k in terms of X_k.