Homework 2

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EE 140: Introduction to Communication Systems

Due : April 3, 2018

Problem 1 (12 points)

For a lowpass signal with a bandwidth of 6000 Hz
(a) What is the minimum sampling frequency for perfect reconstruction of the signal?
(b) What is the minimum required sampling frequency if a guard band of 2000 Hz is required?
(c) What is the minimum required sampling frequency and the value of K for perfect reconstruction if the reconstruction filter has the following frequency response

$$H(f) = \begin{cases} K & |f| < 7000 \\ K - K\frac{|f|-7000}{3000} & 7000 < |f| < 10000 \\ 0 & \text{otherwise} \end{cases}$$

Problem 2 (13 points)

The lowpass signal $x(t)$ with a bandwidth of W is sampled at the Nyquist rate and the signal

$$x_1(t) = \sum_{n=-\infty}^{\infty} (-1)^n x(nT_s)\delta(t - nT_s)$$

is generated. Find the Fourier transform of $x_1(t)$.

Problem 3 (10 points)

Two binary random variables X and Y are distributed according to the joint distribution $p(X = Y = 0) = p(X = 0, Y = 1) = p(X = Y = 1) = \frac{1}{3}$. Compute $H(X), H(Y), H(X|Y), H(X, Y)$ and $I(X; Y)$.

Problem 4 (10 points)

A signal can be modeled as a lowpass stationary process $X(t)$ whose PDF at any time t_0 is given in Figure 1.

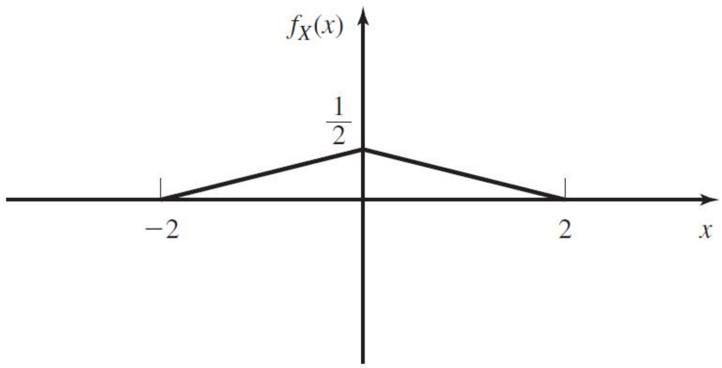

Figure 1

The bandwidth of this process is 5 KHz, and it is desired to transmit it using a PCM system.
(a) If sampling is done at the Nyquist rate and a uniform quantizer with 32 levels is employed, what is the resulting SQNR? What is the resulting bit rate?
(b) If the available bandwidth of the channel is 40 KHz, what is the highest achievable SQNR?

Problem 5 (15 points)

Show that a pulse having the raised cosine spectrum given by Equation (1) satisfies the Nyquist criterion given by Equation (2) for any value of the roll-off factor α.

$$X_{rc}(f) = \begin{cases} T \ , & 0 \leq |f| \leq (1-\alpha)/2T \\ \frac{T}{2}[1+\cos(\frac{\pi T}{\alpha}(|f| - \frac{1-\alpha}{2T}))] \ , & \frac{1-\alpha}{2T} \leq |f| \leq \frac{1+\alpha}{2T} \\ 0 \ , & |f| \geq \frac{1+\alpha}{2T} \end{cases} \tag{1}$$

$$\sum_{m=-\infty}^{\infty} X(f + \frac{m}{T}) = T \tag{2}$$

Problem 6 (15 points)

Binary PAM is used to transmit information over an unequalized linear filter channel. When $a = 1$ is transmitted the noise-free output of the demodulator is

$$x_m = \begin{cases} 0.3 \ , & m = 1 \\ 0.9 \ , & m = 0 \\ 0.3 \ , & m = -1 \\ 0 \ , & \text{otherwise} \end{cases} \tag{3}$$

(a) Design a three-tap zero-forcing linear equalizer so that the output is

$$q_m = \begin{cases} 1 \ , & m = 0 \\ 0 \ , & m = \pm 1 \end{cases} \tag{4}$$

(b) Determine q_m for $m = \pm 2, \pm 3$, by convolving the impulse response of the equalizer with the channel response

Problem 7 (10 points)

Find the capacity of an additive white Gaussian noise channel with a bandwidth of 1 MHz, power of 10W, and noise power-spectral density of $\frac{N_0}{2} = 10^{-9}$ W/Hz.

Problem 8 (15 points)

For the channel shown in Figure 2, find the channel capacity and the input distribution that achieves capacity.

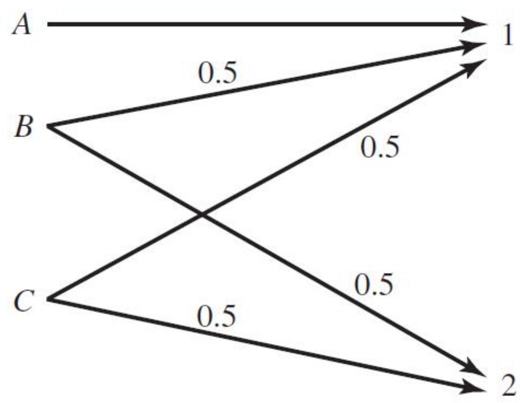

Figure 2