Homework 4

Prof. Xiliang Luo

EE 140: Introduction to Communication Systems

Due Time: May 10, 2018

Problem 1 (20 points)

In an additive white Gaussian noise channel with noise power-spectral density of $\frac{N_0}{2}$, two equiprobable messages are transmitted by

$$s_1(t) = \begin{cases} \frac{At}{T} & 0 \leq t \leq T \\ 0 & otherwise \end{cases}$$

$$s_2(t) = \begin{cases} A(1 - \frac{t}{T}) & 0 \leq t \leq T \\ 0 & otherwise \end{cases}$$

(a) Determine the structure of the optimal receiver.
(b) Determine the probability of error.

Problem 2 (20 points)

Consider a biorthogonal signal set with $M = 8$ signal points. Determine a union bound for the probability of a symbol error as a function of ε_b/N_0. The signal points are equally likely a priori. Where the biorthogonal signal set has the form

$s_1 = [\sqrt{\varepsilon_s}, 0, 0, 0]$ $s_2 = [0, \sqrt{\varepsilon_s}, 0, 0]$ $s_3 = [0, 0, \sqrt{\varepsilon_s}, 0]$ $s_4 = [0, 0, 0, \sqrt{\varepsilon_s}]$
$s_5 = [-\sqrt{\varepsilon_s}, 0, 0, 0]$ $s_6 = [0, -\sqrt{\varepsilon_s}, 0, 0]$ $s_7 = [0, 0, -\sqrt{\varepsilon_s}, 0]$ $s_8 = [0, 0, 0, -\sqrt{\varepsilon_s}]$

Problem 3 (15 points)

The 16-QAM signal constellation shown in Figure 1 is an international standard for telephone-line modems (called V.29). Determine the optimum decision boundaries for the detector, assuming that the SNR is sufficiently high so that errors only occur between adjacent points.

Problem 4 (30 points)

Consider the octal signal-point constellations in Figure 2.
(a) The nearest neighbor signal points in the 8-QAM signal constellation are separated in distance by A units. Determine the radii a and b of the inner and outer circles.

(b) The adjacent signal points in the 8-PSK are separated by a distance of A units. Determine the radius r of the circle.
(c) Determine the average transmitter powers for the two signal constellations and compare the two powers. What is the relative power advantage of one constellation over the other? (Assume that all signal points are equally probable).
(d) Compare the SNR required for the 8-point QAM modulation with that of an 8-point PSK modulation having the same error probability.

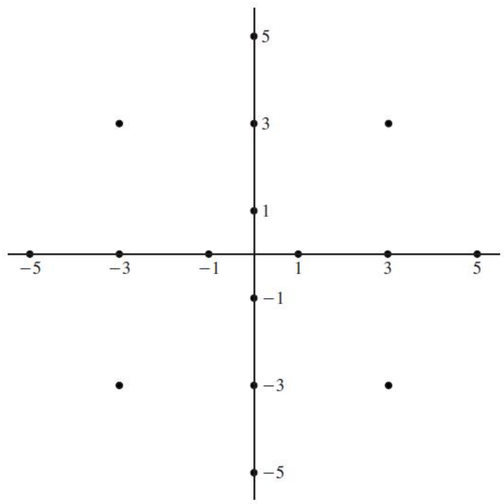

Figure 1

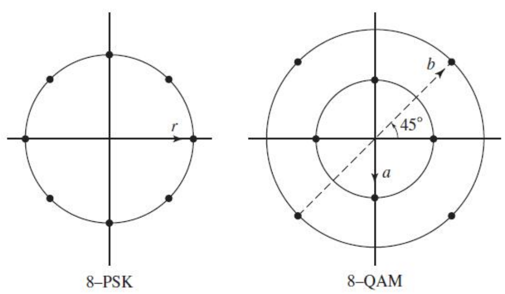

Figure 2

Problem 5 (15 points)

Suppose that the loop filter for a PLL has the transfer function

$$G(s) = \frac{1}{s+\sqrt{2}}$$

(a) Determine the closed-loop transfer function $H(s)$. [Hint:J. Proakis, M. Salehi, Digital Communications, 5th Edition, McGraw-Hill, 2007]
(b) Determine the damping factor and the natural frequency of the loop.