

Numerical Analysis — Homework 1

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Deadline: March 28, 2024

1. What is the definition of the “machine precision” in the context of the IEEE double-precision floating point format?
2. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a three-times continuously differentiable function. Consider the numerical difference formula

$$f'(x) \approx \frac{f(x) - 2f(x + ah) + f(x + bh)}{h}$$

with real-valued coefficients $a, b \in \mathbb{R}$. For which values of a and b is the mathematical approximation error of order $O(h^2)$, i.e., such that

$$\left| f'(x) - \frac{f(x) - 2f(x + ah) + f(x + bh)}{h} \right| = O(h^2) ?$$

3. Plot the numerical evaluation error of the above finite difference formula for $h \in [10^{-16}, 1]$ and $f(x) = \exp(x)$ at $x = 1$ using a logarithmic scale on both axis and interpret your results. For which values of h would you recommend using this formula? You can use any programming language of your choice to complete this exercise.