

# Signal Detection and Parameter Estimation Final Project Report Receiver Cancellation for PA Distortion in MU-MIMO OFDM System

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201210*

**Abstract**—In OFDM system, the combination of several signals with different phases and frequencies that are typical causes a large peak-to-average power ratio (PAPR), which has a nonlinearity distortion in power amplifiers. In this paper, a new technique for power amplifiers nonlinearity cancellation (PANC) is presented in MU-MIMO OFDM system. This technique can reduce the nonlinear distortion effects from the transmitter PA on the received signal and performs well in simulations while the BER has significant improvement. In addition, a novel channel estimation technique that combines frequency channel estimation with PANC is also presented.

**Keywords**—PAPR, Nonlinear distortion, MU-MIMO, OFDM.

## I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) exhibits well-known attributes, such as high bandwidth efficiency and low receiver equalization complexity. OFDM provides high bandwidth efficiency by placing the data and pilot subcarriers with minimal spacing, while still preserving orthogonality among subcarriers. OFDM reduces receiver equalization complexity using lower symbol rates in time combined with a cyclic prefix, while providing higher spectral efficiency and multipath robustness. OFDM efficiently overlaps channels or subcarriers in the frequency domain, which produces a composite time domain signal with a longer symbol time within each of the subcarriers. OFDM's long symbol time, combined with a guard interval length longer than the largest channel or multi-path delay, results in flat fading within in each subcarrier. In this case, the receiver must track only one channel tap per subcarrier, simplifying the receiver equalization complexity. However, synthesizing multiple subcarriers within the same symbol time also has a significant drawback. When multiple carrier frequencies are summed, the peak-to-average power ratio (PAPR) of the composite symbol can be large.

The power amplifier has to provide enough output power for reliable communication. Very often, when analyzing a communication system, it is assumed that the PA is linear, or at least linear within the dynamic range of the transmitted signal. Unfortunately, this is not the case in practice: the PA

distorts the transmitted signal, especially when a high Peak to Average Power Ratio signal is considered. When a multi-cell MIMO system is considered, distortion due to power amplifier becomes a limitation of the accuracy in receiver. Since the PA is an inherently nonlinear device, this output excitation will have an nonlinear impact on the distortion produced by the device.

In this paper, we model the nonlinear distortion as an additional noise component, also referred to as nonlinear distortion noise. In this condition, we introduce an algorithm to reduce the impact of PA nonlinear distortion in MU-MIMO system. The algorithm combined with general detection and estimation can recover the receiver signal accurately with better improvement.

This paper is organized as follows: In section II, we describe the MU-MIMO OFDM system with power amplifier nonlinear distortion. Section III presents the distortion cancellation algorithm. Next in section IV, we propose the whole structure of dealing with the signal in receiver. In Section V, the simulation results are presented. Finally, the paper is terminated with conclusions in section VI.

## II. NONLINEARITY IN MU-MIMO OFDM SYSTEM

We denote the transmitted signal by  $x(n)$ . The output signal of PA  $x^{PA}$  can be denoted as a polynomial function of the input signal as

$$x^{PA}(n) = \sum_{i=1}^L a_i x^i(n) \quad (1)$$

where  $L$  is nonlinearity order of PA, and  $a_i$  are nonlinearity coefficients. Because of frequency selective channels, we assume transmission using OFDM. The same MIMO processing is performed on each subcarrier. Let the MIMO channel of user  $j$  be  $H_j$ ,  $j = 1, 2, \dots, L$ . Then the combined channel matrix is given by

$$H = [H_1^T \ H_2^T \ \dots \ H_L^T]^T$$

The total received vector is given by

$$y_s = [y_1^T \ y_2^T \ \dots \ y_L^T]^T = H_s P_s x_s + H_s D_s + w_s \quad (2)$$

$D_s = [D_1 \ D_2 \ \dots \ D_L]$  is the PA distortion in transmitter.  $y_j$  represents the received signal vector at the  $j$ th user as

$$y_j = H_j(P_j x_j + D_j) + H_j\left(\sum_{l=1, l \neq j}^L P_l x_l + D_l\right) + w_j \quad (3)$$

Next, denoting the overall receive filter

$$M_s = \text{diag}\{M_1, M_2, \dots, M_L\}$$

where  $M_j$  represents the  $j$ th user's receiver filter, the receive filter output vector of the  $j$ th user  $x_j$  can be written as

$$x_j = M_j H_j(P_j x_j + D_j) + M_j H_j\left(\sum_{l=1, l \neq j}^L P_l x_l + D_l\right) + M_j w_j. \quad (4)$$

### III. POWER AMPLIFIERS NONLINEARITY CANCELLATION

Assuming that the transmitter nonlinearity is known at the receiver, the receiver can compute and estimate  $\hat{d}(n)$  from the received vector  $x(n)$ . An initial estimate of vector  $\hat{x}(n)$  can be used to calculate the distortion vector  $d(n)$ . The estimated distortion vector is removed from the original received vector, and a new estimation of  $x(n)$  can be obtained.

Assuming that the NL model of the PA  $g[\cdot]$ , is known at the receiver (the PA model is discussed in Appendix), we can get the distortion by

$$\hat{d}(n) = g[\hat{x}^m] - K_L \hat{x}^m(n) \quad (5)$$

where  $m$  denote the iteration time. After the FFT operation,

$$\hat{d}(n) = Q_N^H \{g[\hat{x}^m(n)] - K_L \hat{x}^m(n)\} \quad (6)$$

Using this result, the transmitted constellation is re-estimated in a new distortion cancellation step.

### IV. CHANNEL ESTIMATION WITH PANC

In general, there are two approaches for channel estimation. One approach assumes the receiver has received the prior information of the whole OFDM symbol. The other way is to take advantage of a reduced set of dedicated subcarriers for pilot data and estimate channel by interpolation. In this paper, the second approach is taken to estimate the whole channel.

Using the MMSE detector, the estimation is given by

$$\hat{x}(n) = \frac{1}{K_L} [H(n)H^H(n, k) + \sigma_n^2 I]^{-1} H(n) \quad (7)$$

According to the method to estimate the channel, we can design the algorithm as follows.

In the algorithm,  $m$  denote the iteration time,  $i$  and  $j$  denote the number of receive antennas and the number of users.

### Algorithm 1 CHANNEL ESTIMATION WITH THE PANC ALGORITHM

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1: function PANC_ESTIMATION( $H$ )
2:   for  $k = 1 \rightarrow N$  do
3:      $W_{MMSE} = \frac{1}{K_L} [H(n, k)H^H(n, k) + \sigma_n^2 I]^{-1} H(n, k)$ 
4:      $\hat{x}(n, k) = W_{MMSE}^H y(n, k)$ 
5:   end for
6:   for  $j = 1 \rightarrow L$  do
7:     for  $i = 1 \rightarrow P$  do
8:        $\hat{d}_j(n) = g[x_j^m(n)] - K_L x_j^m(n)$ 
9:        $\hat{d}_j(n) = Q^H \hat{d}_j(n)$ 
10:       $\hat{h}_{i,j}(n, k) = \frac{y_i(n, k)}{K_L x_j(n, k) + \hat{d}_j(n, k)}$ 
11:       $\hat{h}_{i,j}(n) = Q_N [Q_T^H Q_T]^{-1} Q_T^H \hat{h}_{i,j}^c(n)$ 
12:    end for
13:  end for
14:  for  $j = 1 \rightarrow L$  do
15:    for  $i = 1 \rightarrow P$  do
16:       $\bar{y}_i(n) = y_i(n) - Q_N \sum_{j=1; j \neq i}^L \text{diag}[\hat{h}_{i,j}(n)] [z_j(n) + \hat{d}_j(n)]$ 
17:       $\hat{h}_{i,j}(n) = [C^H(n)C(n)]^{-1} C^H(n) \bar{y}_i(n)$ 
18:    end for
19:  end for
20: end function

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### V. SIMULATION RESULTS

The algorithm is simulated in MU-MIMO OFDM system with two users, two transmitted antennas and two received antennas. The environment is based on Wi-Fi indoor conditions which contains 9 taps and the communication distance is less than 10m. The modulation type is QPSK. The number of subcarriers employed is  $N = 256$ . Since for practical wireless systems only a limited number of odd orders of polynomial contribute to the PA nonlinearity distortion and the higher orders could be neglected, the actual nonlinear behavior of PA can be modeled simply as

$$f(x) = a_1 x + a_2 |x|^2 x + a_3 |x|^4 x \quad (8)$$

where  $x$  is the input signal of PA, and  $y$  is the output signal of PA.

In simulation, we assume the actual PA nonlinearity coefficients as  $a_1 = 1.0513 + 0.0904i$ ,  $a_2 = -0.0542 - 0.2900i$ ,  $a_3 = -0.9657 - 0.7028i$ . And for the estimated PA model, only the third-order nonlinearity coefficient is estimated which is a common assumption in literature. Thus the nonlinearity coefficients estimation is limited by the estimated model accuracy, and then the estimation error for the nonlinearity coefficients is unavoidable in practice. From the curve in figure 1, we can find the PANC has a good performance compared with only MMSE. The algorithm reduces the impact of PA distortion in receiver.

### VI. CONCLUSION

This paper studied power amplifier nonlinear distortion effects in spatial modulation systems. The nonlinear effects at

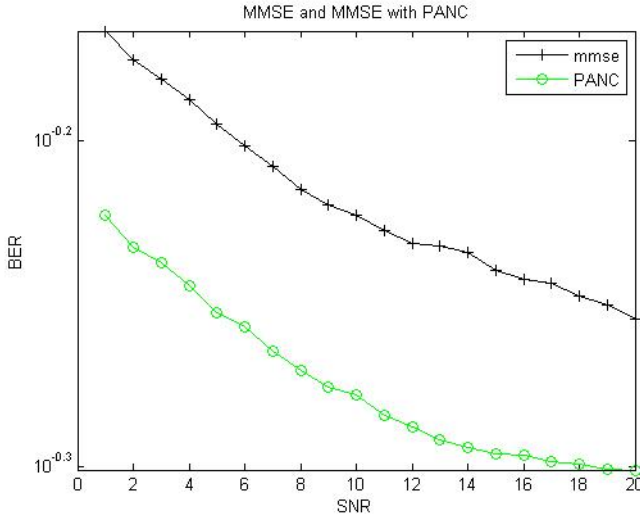


Fig. 1. The performance of MMSE and MMSE with PANC

low input backoffs significantly distorted the signal constellation, which resulted in BER degradation. In this paper, we have proposed a method by PANC algorithm to cancel the NL distortion effect in MU-MIMO system. Simulation results and analysis show that the estimation performance has improved with the method.

#### APPENDIX A PA MODULE

The PA input signal can be represented in polar coordinates as  $x = \beta e^{j\phi}$ , and the output of the PA can be written as

$$g[x] = M[\beta] \exp(j(\phi + P[\beta]))$$

where  $M[\beta]$  represents the AM/AM conversion, and  $P[\beta]$  represents the AM/PM conversion characteristics of the PA. The output of the PA can be written as

$$g[x] = K_L x + d$$

where  $K_L$  is a complex scalar that defines the linear gain, and  $d$  is an uncorrelated distortion term. Thus, we have

$$E[xd^*] = E[x(g[x] - K_L x)^*] = 0$$

from which we obtain the gain as

$$K_L = \frac{E[xg^*[x]]}{E[xx^*]}$$

The NL distortion term can be calculated as

$$\begin{aligned} \sigma_d^2 &= \frac{E[|g[x]|^2] - |K_L|^2}{E[|x|^2]} \\ &= \frac{\int_0^{+\infty} |g[u]|^2 p(u) du - |K_L|^2}{\sigma^2} \end{aligned}$$

where  $g[u] = M_s(\beta)$  is the PA transfer function, and  $p(u)$  is the pdf of the OFDM signal, which, for large  $N$ , take the form

$p(u) = \left(\frac{1}{\sqrt{2\pi}\sigma}\right) \exp(-u^2/2\sigma^2)$ . The PA transfer characteristic is modeled by

$$M_S(\beta) = \frac{\beta}{[1 + (\beta/A_s)^{2p}]^{1/2p}}$$

The limited model is

$$M_L(\beta) = \begin{cases} \beta, & \beta < A_s \\ A_s, & \beta > A_s \end{cases}$$

where  $A_s$  is the amplifier input saturation voltage. Using the limiter model, it is possible to obtain closed-form solutions for the  $K_L$  and  $\sigma_d^2$  values. We can get the following expression for  $K_L$ :

$$K_L = 1 - \exp(-v^2) + \frac{1}{2} \sqrt{\pi} \operatorname{erfc}(v)$$

where  $v$  is the clipping level. The power of the distortion noise  $\sigma_d^2$  is

$$\begin{aligned} \sigma_d^2 &= \frac{\int_0^{+\infty} |g[u]|^2 p(u) du - K_L^2}{\sigma^2} \\ &= \frac{\int_0^{A_s} |g[u]|^2 p(u) du + \int_{A_s}^{+\infty} |A_s|^2 p(u) du - K_L^2}{\sigma^2} \\ &= \sigma^2 [1 - \exp(-v^2) - K_L^2]. \end{aligned}$$

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