The detection of the yellow ball by using Multi-dimensional Gaussian distribution and EM algorithm

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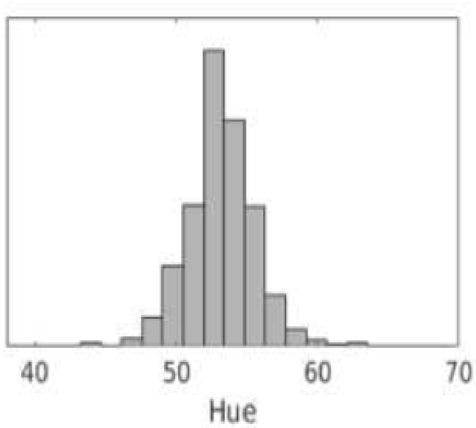
Abstract—Suppose there is a soccer robot,and the yellow ball is the "soccer",and there are two kinds of balls,yellow and red.It's very easy for our human to distinguish which one is yellow,but it's very difficult for a robot because the input to the robot is pixel.It needs to create a mapping from the value of the pixel to "yellow" or "red".And according to the central limit theorem,when the n is large,most of the distribution will converge to Gaussian distribution.And then we can use Expectation Maximization algorithm to calculate the mean and the covariance.

Index Terms—Matlab simulation,Gaussian distribution,EM algorithm;

1 INTRODUCTION

Suppose there is a soccer robot,and the yellow ball is the "soccer",and there are two kinds of balls,yellow and red.It's very easy for our human to distinguish which one is yellow,but it's very difficult for a robot because the input to the robot is pixel.However,the pixel is different.We can download a picture about the Hue value a yellow ball by baidu search engine.



from the above picture,we can see that the yellow can have a lot of Hue value,and the shape of the distribution is the same with Gaussian distribution.

The first method to distinguish the yellow ball is save all the Hue value,and then compare the pixel with the Hue value，however this method will waste a lot of memory.The second method is using the Gaussian distribution,we need to calculate the mean and the variance.And we can use the two values to save all the Hue value approximately.For one-dimensional Gaussian distribution,we can use the Maximum Likelihood Estimation to calculate the mean and variance.For Multi-dimensional Gaussian distribution,using the MLE is very difficult,so we need to use EM algorithm to calculate approximately [1] [3]. Jan15, 2018

2 GAUSSIAN DISTRIBUTION

2.1 one-dimensional Gaussian distribution

as we know,the mathematical expressions of the Gaussian distribution $x \sim N(\mu, \sigma^2)$ is $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ and the mean is μ,variance is

σ^2.When the mean changes,the graph just pans left and right.And when the variance change,the graph changes up and down.

2.2 Maximum Likelihood Estimation

suppose we have the pdf $p(x|\mu,\sigma)$ about parameter μ,σ;and we have a total n observation,the sample is $X = X_1, ..., X_n$ which is identity independent distribution.and $x_1, ..., x_n$ is the observation [1] [2] [3].so that the joint probability density function is

$$p(X|\mu,\sigma) = \prod_{i=1}^{n} p(x_i|\mu,\sigma) \tag{1}$$

It is also called Likelihood function.And our goal is to get the $\hat{\mu}$ and $\hat{\sigma}$ which make the Likelihood function biggest when given observation X_i so

$$\hat{\mu},\hat{\sigma} = max\prod_{i=1}^{n} p(x_i|\mu,\sigma) = max\prod_{i=1}^{n} lnp(x_i|\mu,\sigma) \tag{2}$$

and

$$\begin{aligned} lnp(x_i|\mu,\sigma) &= ln(\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}) \\ &= -\frac{(x_i-\mu)^2}{2\sigma^2} - ln\sigma - ln\sqrt{2\pi} \end{aligned}$$

thus,we can get that

$$\begin{aligned} \hat{\mu},\hat{\sigma} &= max\prod_{i=1}^{n} lnp(x_i|\mu,\sigma) \\ &= max\sum_{i=1}^{n} -\frac{(x_i-\mu)^2}{2\sigma^2} - ln\sigma \\ &= min\sum_{i=1}^{n} \frac{(x_i-\mu)^2}{2\sigma^2} + ln\sigma \end{aligned} \tag{3}$$

and let $L(\mu,\sigma) = \sum_{i=1}^{n} \frac{(x_i-\mu)^2}{2\sigma^2} + ln\sigma$

In order to get the minimal L,we can seek the partial derivative about μ and σ,so

$$\frac{\alpha L}{\alpha\mu} = \sum_{i=1}^{n} \frac{-2(x_i-\mu)}{2\sigma^2} = \frac{1}{\sigma^2}(n\mu - \sum_{i=1}^{n} x_i) = 0 \tag{4}$$

so $\hat{\mu} = \frac{1}{N}\sum_{i=1}^{N} x_i$;similarly,

$$\frac{\alpha L}{\alpha\sigma} = \frac{1}{\sigma}(N - \frac{1}{\sigma^2}\sum_{i=1}^{N}(x_i-\hat{\mu})^2) = 0 \tag{5}$$

so $\hat{\sigma}^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i-\hat{\mu})^2$

we can use the MLE calculate the sample mean and variance.

2.3 Multi-dimensional Gaussian distribution

Under one-dimensional Gaussian distribution,we just define Hue value,but the color is defined by multi-dimension such as under HSV model,there are Hue,Saturation and Value.In order to solve this problem, we need to use Multi-dimensional Gaussian distribution.and it's mathematical expressions is

$p(x) = \frac{1}{(2\pi)^{D/2}|\varepsilon|^{1/2}}exp-\frac{1}{2}(x-\mu)^T\varepsilon^{-1}(x-\mu)$

where x is a vector,D is the dimension ,μ is mean vector,ε is the Covariance matrix

2.4 MIE in Multi-dimensional Gaussian distribution

similar to the one dimension Gaussian distribution,

$$\begin{aligned} &\hat{\mu},\hat{\varepsilon} = argmaxp(x_i|\mu,\varepsilon) \\ &= argmin\sum_{i=1}^{N}(\frac{1}{2}(x_i-\mu)^T\varepsilon^{-1}(x_i-\mu) + \frac{1}{2}ln|\varepsilon|) \end{aligned}$$

let $L = argmin\sum_{i=1}^{N}(\frac{1}{2}(x_i-\mu)^T\varepsilon^{-1}(x_i-\mu) + \frac{1}{2}ln|\varepsilon|)$

and calculate partial derivative on μ and ε

then ,we can get that

$\hat{\mu} = \frac{1}{N}\sum_{i=1}^{N} x_i$;

$\hat{\varepsilon} = \frac{1}{N}\sum_{i=1}^{N}(x_i-\hat{\mu})(x_i-\hat{\mu})^T$

2.5 Gaussian mixture distribution

Gaussian mixture distribution is the sum of a lot of single Gaussian distribution,and almost any distribution can be showed by GMM.In the detection of robot,we can found that the observation of the red ball is not very good in RGB model,so we will use a mixture of two Gaussian.

The mathematical expressions is

$$p(x) = \sum_{k=1}^{N} w_k g_k(x|\mu_k, \varepsilon_k)$$

where $g_k \sim N(\mu_k, \varepsilon_k)$,and w_k is a gain $(\sum_{k=1}^{N} w_k = 1)$

2.6 EM algorithm calculate the mean and covariance

In single Gaussian distribution, we can use MLE to calculate the mean and variance,but in the GMM, it is very difficult.So we use EM to do it.First we choose a initial solution$\mu_1, \varepsilon_1, ..., \mu_k, \varepsilon_k$ second,we calculate

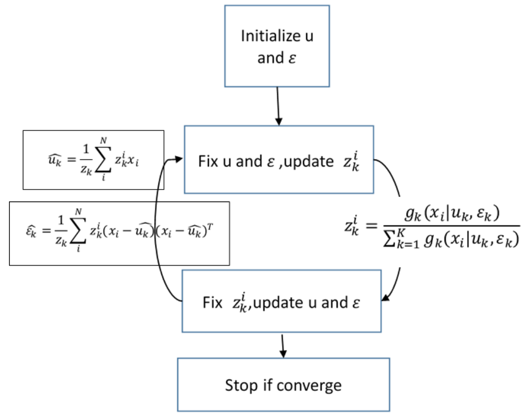
$$z_k^i = \frac{g_k(x_i|\mu_k, \varepsilon_k)}{\sum_{i=1}^{K} g_k(x_i|u_k, \varepsilon_k)} (k = 1, 2, ..., K; i = 1, 2, ..., N)$$

where K is the number of single Gaussian distribution,N is the number of observation.Then we can use the z_k^i to update the μ_k and ε_k,

$$\mu_k = \frac{1}{z_k} \sum_{i=1}^{N} z_k^i x_i;$$

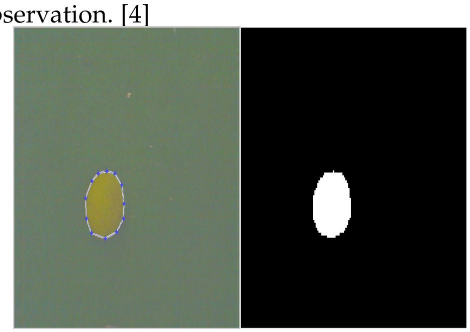
$$\varepsilon_k = \frac{1}{z_k} \sum_{i=1}^{N} z_k^i (x_i - \mu_k)(x_i - \mu_k)^T$$

the flow chart about EM algorithm is

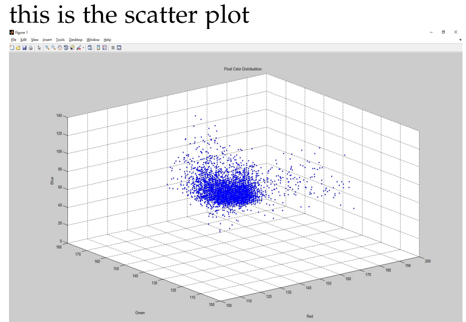


3 SIMULATION IN MATLAB

First,construct the model of color from the picture of course "Robotics: Estimation and Learning".And we need to mark the location of the yellow ball by hand.Then we can get a lot of observation. [4]



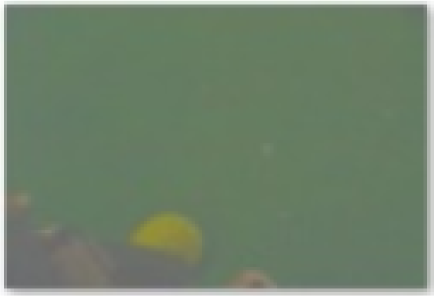
this is the scatter plot

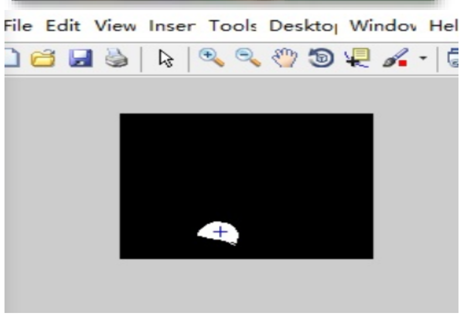


then ,we us the EM algorithm calculate the mean μ and covariance matrix ε,and save it.

after that ,we need to detect the yellow ball by calculate whether the pixel is belongs to the yellow ball.

the result is :





4 Conclusion

By doing this project,I have deeply understood the Gaussian distribution and the EM algorithm.And I have feeled the power of MLE.And I found the deep learning is very useful and interesting.

What's more,after construct the model,any input in this 19 picture,the matlab can output the location of the yellow ball approximately.

References

[1] 张宏东*EM*算法及其应用,山东大学，2014-5-7.

[2] Steven M.Key*Fundamentals of Statistical Signal Processing*，电子工业出版社，2006.7

[3] 江 山,当 机 器 人 遇 上 了 学 习,2017 https://zhuanlan.zhihu.com/p/21648507

[4] Daniel LeeProfessor of Electrical and Systems Engineering*course:Robotics: Estimation and Learning* https://www.coursera.org/learn/robotics-learning