A SIMPLE PAYOFF ALLOCATION MECHANISM ACHIEVES EFFICIENCY AND STABILITY IN RENEWABLE ENERGY AGGREGATION

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ABSTRACT

Aggregating statistically diverse renewable power producers (RPPs) is an effective way to reduce the uncertainty of the RPPs. The key question is how to design a mechanism to aggregate the RPPs and distribute the payoffs among them. In this paper, a simple payoff allocation mechanism (PAM) is shown to achieve a wide range of desired properties. In particular, social efficiency, stability (in the core), and no collusion are achieved at the unique pure Nash Equilibrium (NE) of the non-cooperative game of RPPs induced by the PAM. As a result, an ideal "Price of Anarchy" of one is achieved. Moreover, a closed form expression of the unique pure NE is derived. A simulation study is conducted using the data of ten wind power producers in the PJM interconnection.

***Index Terms*—** Mechanism Design, Renewable Energy Integration, Energy Aggregation

1. INTRODUCTION

Renewable energies play a central role in achieving a sustainable energy future. However, renewable energies such as wind and solar power are inherently uncertain and variable, and integrating them into the power system raises significant reliability and efficiency challenges [1, 2]. A variety of approaches have been proposed to compensate for the uncertainty of renewable energies, such as improving renewable power generation prediction [2], employing better generation dispatch methods [3], fast-ramping generators, energy storage control [4, 5, 6], and demand response programs [7, 8, 9].

Another solution that has received considerable attention is to aggregate statistically diverse renewable energy sources [1, 10, 11]. In an aggregation, renewable power producers (RPPs) pool their generation together so as to reduce the aggregate uncertainty and risk. As a result, by forming an aggregation, the RPPs can in total earn a higher payoff. A central question in aggregating RPPs is thus how to distribute the total payoff of an aggregation among its member RPPs.

Notably, aggregating renewable energies has been studied extensively in the context of a two-settlement power market model, consisting of a forward power market and a real time one. Based on the *joint probability distribution* of renewable energies, aggregating RPPs and allocating payoffs have been studied in a *coalitional game* framework [10, 11]. The primary interest in this setting is to find a payoff allocation solution that is *in the core* of the game, which is in general an NP hard problem. The core is proven to be non-empty in [10], and a closed form solution in the core is found in [11].

While this line of works achieve efficiency (optimality) and stability (in the core) in aggregating RPPs, an underlying assumption is that the aggregator knows the joint probability distribution of the RPPs' generation. Relaxing this assumption, another line of works have studied payoff allocation solely based on *realized* generation of RPPs. Notably, a simple interface between aggregator and RPPs has been proposed in [12]: each RPP submits a number to the aggregator, and the aggregator simply passes on the sum of these numbers as the forward power contract. Based on this interface, a number of payoff allocation mechanisms (PAMs) have been proposed [12, 13, 14, 15]. Given any PAM, the RPPs' decision making at the time of forward power market entails a *non-cooperative game* (as will be described later in Section 2.3), and properties of the Nash Equilibria of this game have been studied in these works. The existing PAMs in the literature, however, have only gained limited success, as some essential properties which are highly desired still cannot be achieved. In particular, achieving efficiency and stability at the Nash Equilibria remains to be open questions.

In this paper, we employ the above simple interface between aggregator and RPPs. Surprisingly, we show that a simple PAM can in fact achieve all the essential properties that we desire. In particular, efficiency (optimality), individual rationality, stability (in the core), and no collusion can all be achieved in the unique pure Nash Equilibrium (NE) of the non-cooperative game of RPPs given the proposed PAM. Moreover, this unique NE enjoys a *closed form* expression.

The remainder of the paper is organized as follows. The problem is formulated in Section 2. The design goals, i.e., the desired properties of the PAM are described in Section 3. The main results are presented in Section 4. Simulations are conducted in Section 5. Conclusions are drawn in Section 6. Due to space limitations, proofs of the theorems are left out and can be found in [16].

2. PROBLEM FORMULATION

2.1. Renewable Power Producers in a Two Settlement Power Market

We consider RPPs participating in a two-settlement power market consisting of a day-ahead (DA) and a real time (RT) market. As a baseline case, we consider an RPP i who participates in the market *separately* from the other RPPs. In the DA market, RPP i's generation at the time of interest in the next day is modeled as a random variable, denoted by X_i. RPP i then sells a forward power supply contract in the amount of c_i in the DA-market. Interchangabely, c_i is also termed a day-ahead commitment. RPP i gets a payoff of $p^f c_i$ where p^f denotes the price in the DA market. At the delivery time in the next day, RPP i obtains its realized generation x_i: a) If it faces a shortfall, i.e., $c_i - x_i > 0$, it needs to purchase the remaining power from the RT market at a real-time buying price $p^{r,b}$, b) if it has excess power, i.e. $x_i - c_i > 0$, it can sell it in the RT-market at a real-time selling price $p^{r,s}$. In case excess power needs to be penalized as opposed to rewarded, we model such cases by having $p^{r,s} < 0$. The only assumption we make on prices is $p^{r,s} \leqslant p^{r,b}$, which must hold for no arbitrage.

As a result, the *realized* payoff of an RPP i who separately participates in the two-settlement market is given by

$$\mathcal{P}_i^{sep} = p^f c_i - p^{r,b} (c_i - x_i)_+ + p^{r,s} (x_i - c_i)_+ \quad (1)$$

where $(\cdot)_+ = \max(0, \cdot)$. We denote the expected payoff of RPP i at the time when c_i is determined one day ahead by

$$\pi_i^{sep}(c_i) = \mathbb{E}[\mathcal{P}_i^{sep}], \quad (2)$$

where the expectation is taken over the random renewable power generation X_i.

2.2. Aggregation of RPPs and Payoff Allocation

We consider an aggregator that a) aggregates the power generation from a set of N RPPs, denoted by $\mathcal{N}$, b) participates in the DA-RT market on behalf of them, and c) allocates its payoff back among the RPPs. As a design choice, we consider the following interface between an aggregator and RPPs (cf. [12, 13, 14, 15]):

a. In the DA market, each RPP i submits a DA commitment c_i to the aggregator, and the aggregator sells a forward power contract in the amount of $c_{\mathcal{N}} = \sum_{i\in\mathcal{N}} c_i$.

b. At the delivery time, the aggregator collects all the realized generation from the RPPs, denoted by $x_{\mathcal{N}} = \sum_{i\in\mathcal{N}} x_i$, to meet the commitment $c_{\mathcal{N}}$. The deviation is settled in the RT market in the same way as in Section 2.1. The realized payoff of the aggregator is thus

$$\mathcal{P}_{\mathcal{N}} = p^f c_{\mathcal{N}} - p^{r,b}(c_{\mathcal{N}} - x_{\mathcal{N}})_+ + p^{r,s}(x_{\mathcal{N}} - c_{\mathcal{N}})_+ \quad (3)$$

c. The aggregator returns a payoff of $\mathcal{P}_i$ to each RPP i.

The key design question that remains for the aggregator is how to determine the payoffs $\mathcal{P}_i$.

2.3. Decisions of RPPs in a Non-Cooperative Game

Given any PAM that specifies the rule of determining $\mathcal{P}_i$, an RPP i is free to submit any DA commitment c_i to the aggregator. In particular, a rational RPP i would submit a c_i that *maximizes* its expected payoff $\mathbb{E}[\mathcal{P}_i]$. Note that, given a PAM, $\mathbb{E}[\mathcal{P}_i]$ can also depend on the *other* RPPs' submissions of commitments. Accordingly, we denote this expected payoff by $\pi_i(c_i, \{c_{-i}\})$, where $\{c_{-i}\}$ denotes the set of commitments of the RPPs other than i. Note that $\pi_i(c_i, \{c_{-i}\})$ depends on the particular PAM employed.

Therefore, the decisions of the N RPPs on their submissions $\{c_i\}$ can be studied under a *non-cooperative game* framework, (termed a "contract game" in [12].) As a mechanism designer for aggregating RPPs, we are interested in designing a PAM so that certain desirable properties can be achieved *at equilibria* of this non-cooperative game given the designed PAM.

3. DESIGN GOALS: DESIRED EFFICIENCY AND STABILITY PROPERTIES

In this section, we provide a detailed description of the desired properties in designing PAM.

3.1. Ex-Ante Properties

We first describe the desired properties of the expected payoffs *when RPPs submit* $\{c_i\}$ *one day ahead of delivery*. We refer to these as "ex-ante" properties.

1. *Existence and uniqueness of pure Nash equilibrium (NE)*: Given a PAM, an NE is a set of commitments $\{c_i^\star\}$ that satisfy $\forall i, c_i^\star \in \operatorname{argmax}_{c_i} \pi_i\left(c_i, \{c_{-i}^\star\}\right)$.

2. *Efficient computation of NE*: In particular, we are interested in whether the NE can be computed in closed form.

3. *Efficiency*: A PAM is efficient if, at the NE, the aggregation achieves the maximum expected payoff for the entire group of RPPs. Specifically, this means that $\sum_{i=1}^N c_i^\star$ is equal to the optimal commitment for the entire aggregation $c_{\mathcal{N}}^\star = \operatorname{argmax}_{c_{\mathcal{N}}} \mathbb{E}[\mathcal{P}_{\mathcal{N}}]$ (cf. (3)). This optimal commitment can in fact be computed as a solution to a news-vendor problem [17]: $c_{\mathcal{N}}^\star = \mathcal{F}_{\mathcal{N}}^{-1}\left(\frac{p^f - p^{r,s}}{p^{r,b} - p^{r,s}}\right)$, where $\mathcal{F}_{\mathcal{N}}(x_{\mathcal{N}})$ is the cumulative distribution function of the *aggregate* generation $X_{\mathcal{N}} = \sum_{i=1}^N X_i$.

4. *Ex-ante individual rationality*: At the NE, the expected payoff of RPP i should be at least as high as the *maximum* payoff it could have gotten had it separately participated in the DA-RT market. Specifically,

$$\forall i \in \mathcal{N}, \quad \pi_i\left(c_i^\star, \{c_{-i}^\star\}\right) \geqslant \pi_i^{sep}\left(c_i^{\star,sep}\right), \qquad (4)$$

where $c_i^{\star,sep} = \operatorname{argmax}_{c_i} \pi_i^{sep}(c_i)$ (cf. (2)). Notably, the (separately) optimal commitment $c_i^{\star,sep}$ is in general *not equal* to the equilibrium commitment $c_i^\star$.

5. *Ex-ante in the core*: As a generalization of individual rationality to a much stronger sense of stability, being in the core in an "ex-ante" sense means that the RPPs' expected payoffs satisfy the following condition: if any subset $\mathcal{T}$ of the RPPs leave the aggregation, separately form their own aggregation, and then participate in the market, their highest possible expected payoff would be no higher than the sum of their expected payoffs originally from the PAM at the NE. Specifically,

$$\forall \mathcal{T} \subset \mathcal{N}, \quad \sum_{i\in\mathcal{T}} \pi_i\left(c_i^\star, \{c_{-i}^\star\}\right) \geqslant \pi_{\mathcal{T}}^{sep}\left(c_{\mathcal{T}}^{\star,sep}\right), \qquad (5)$$

where

$$\pi_{\mathcal{T}}^{sep}(c_{\mathcal{T}}) = \mathbb{E}\big[p^f c_{\mathcal{T}} - p^{r,b}\left(c_{\mathcal{T}} - x_{\mathcal{T}}\right)_+ \qquad (6)$$
$$+ p^{r,s}\left(x_{\mathcal{T}} - c_{\mathcal{T}}\right)_+\big], \qquad (7)$$

and $c_{\mathcal{T}}^{\star,sep} = \operatorname{argmax}_{c_{\mathcal{T}}} \pi_{\mathcal{T}}^{sep}(c_{\mathcal{T}})$.

6. *Ex-ante no collusion*: Suppose a subset of RPPs join together as a *single player* before participating in the aggregation with the remaining RPPs. Because of the change of the set of players, a new NE would arise in this new game. The expected payoff of this "joint player" at this new NE should be *no higher* than the sum of these RPPs' expected payoffs at the NE of the original game. Otherwise, some RPPs could have incentives to collude, join together, and collectively interface with the aggregator as a single (and larger) RPP in order to earn a higher total payoff.

3.2. Ex-Post Properties

We now describe the desired properties of the realized payoffs of the RPPs after the realized renewable generation are revealed and the imbalances settled at RT. When a property is achieved for *all possible realizations* of renewable power generation, we refer to it as an "ex-post" property. The following are all "ex-post" properties in this sense.

1. *Ex-post budget balance*: $\sum_{i\in\mathcal{N}} \mathcal{P}_i = \mathcal{P}_{\mathcal{N}}$.

2. *Ex-post restricted individual rationality*: $\mathcal{P}_i \geqslant \mathcal{P}_i^{sep}$, where $\mathcal{P}_i^{sep}$ is the realized payoff of RPP i had it separately participated in the DA-RT market *with the same DA commitment c_i as originally submitted to the aggregator*.

3. *Ex-post restrictedly in the core*: Being (restrictely) in the core in an "ex-post" sense means the RPPs' realized payoffs satisfy the following condition: if any subset of the RPPs, denoted by $\mathcal{T}$, leave the aggregation, separately form their own aggregation, and then participate in the market *with the same sum of DA commitments $\sum_{i\in\mathcal{T}} c_i$ as originally submitted to the aggregator*, they will get a realized payoff no higher than the sum of their realized payoffs originally from the PAM.

4. *Ex-post restricted no collusion*: Suppose a subset of RPPs, denoted by $\mathcal{T}$, join together as a single player before participating in the aggregation with the remaining RPPs, and submit the same sum of DA commitments $\sum_{i\in\mathcal{T}} c_i$ to the aggregator. Their total realized payoff should be *no higher* than the sum of their original realized payoffs from the PAM.

5. *Fairness*: For any two RPP_i and RPP_j, if $c_i - x_i = c_j - x_j$, then $\mathcal{P}_i - p^f c_i = \mathcal{P}_j - p^f c_j$.

6. *No-exploitation*: If $c_i - x_i = 0$, then $\mathcal{P}_i = p^f c_i$.

Remark 1 *It is important to note that we call the ex-post properties 2., 3., and 4. "restricted" ones. The restriction lies in the requirement that the same DA commitments $\{c_i\}$ submitted to the aggregation must continue to be used by any RPPs that leave the aggregation. In other words, leaving RPPs are not allowed to re-adjust their DA commitments.*

4. MAIN RESULTS

We now introduce the following payoff allocation mechanism, developed in our earlier work [18]:

$$\mathcal{P}_i = \begin{cases} p^f c_i + p^{r,b}\left(x_i - c_i\right) & \text{if } x_{\mathcal{N}} - c_{\mathcal{N}} < 0 \\ p^f c_i + p^*\left(x_i - c_i\right) & \text{if } x_{\mathcal{N}} - c_{\mathcal{N}} = 0 \\ p^f c_i + p^{r,s}\left(x_i - c_i\right) & \text{if } x_{\mathcal{N}} - c_{\mathcal{N}} > 0 \end{cases}, \qquad (8)$$

where $p^{r,s} \leqslant p^* \leqslant p^{r,b}$, and p^* can be chosen arbitrarily within this range.

Given this PAM, we show that the non-cooperative game among the RPPs possesses a unique pure NE, which moreover can be computed in closed form.

Theorem 1 *Given the PAM* (8)*, the following DA commitments give the unique pure NE of the non-cooperative game of the RPPs submitting $\{c_i\}$ (cf. Section 2.3): $\forall i = 1, \ldots, N$,*

$$c_i^\star = \mathbb{E}\left[X_i \middle| X_{\mathcal{N}} = c_{\mathcal{N}}^\star\right], \qquad (9)$$

where $X_{\mathcal{N}} = \sum_{i=1}^{N} X_i$, $c_{\mathcal{N}}^\star = \mathcal{F}_{\mathcal{N}}^{-1}\left(\frac{p^f - p^{r,s}}{p^{r,b} - p^{r,s}}\right)$, and $\mathcal{F}_{\mathcal{N}}\left(x_{\mathcal{N}}\right)$ is the cumulative distribution function of $X_{\mathcal{N}}$.

Theorem 1 immediately implies the following:

Corollary 1 *Given the PAM* (8)*, social efficiency is achieved at the NE, i.e.,* $\sum_{i=1}^{N} c_i^\star = c_\mathcal{N}^\star = \mathrm{argmax}_{c_\mathcal{N}} \mathbb{E}[\mathcal{P}_\mathcal{N}]$.

Furthermore, we show that the other desired properties introduced in Section 3.1 are also achieved.

Theorem 2 *Given the PAM* (8)*, ex-ante individual rationality, ex-ante in the core, and ex-ante no collusion are achieved at the unique pure NE specified in* (9).

We note that ex-ante in the core implies individual rationality. As a result of Theorem 1, 2 and Corollary 1, we conclude that the proposed PAM (8) induces a *unique stable NE* among the RPPs, expressed in *closed form* (9), which is moreover economically *efficient* for the entire aggregation. In particular, the proposed PAM achieves an ideal "Price of Anarchy" of one (cf. [19]).

Lastly, it has been shown in our earlier work [18] that the proposed PAM also achieves all the ex-post properties introduced in Section 3.2, regardless of what DA commitments $\{c_i\}$ the RPPs submit.

5. SIMULATION

5.1. Data Description and Simulation Setup

We perform simulations using the NREL dataset [20] based on ten wind power producers (WPPs) located in PJM for the month of Feb. 2004. We compare the following cases in which WPPs earn payoffs in different ways:

- Case 1: The aggregator employs a previously developed efficient and ex-ante in the core PAM to aggregate the WPPs [11]. This PAM is based on the knowledge of the joint probability distribution of the WPPs' random generation.

- Case 2: The aggregator employs the proposed PAM (8) to aggregate the WPPs. Each WPP i submits $c_i^{\star,sep} = \mathrm{argmax}_{c_i} \pi_i^{sep}(c_i)$, which can be solved as a newsvendor problem [17].

- Case 3: The aggregator employs the proposed PAM (8) to aggregate the WPPs. Each WPP i submits $c_i^\star$ (cf. (9)), i.e., its decision at the unique NE.

- Case 4: Each WPP i separately participates in the DA-RT market, and makes the optimal DA commitment $c_i^{\star,sep} = \mathrm{argmax}_{c_i} \pi_i^{sep}(c_i)$.

5.2. Simulation and Results

Daily average payoff of the WPPs for the four cases are shown in Figure 1. We note that WPP aggregation (Case 1-3) consistently give (on average) higher payoffs to all the WPPs comparing to not aggregating (Case 4). This highlights the benefit of aggregation in increasing the payoffs of the WPPs.

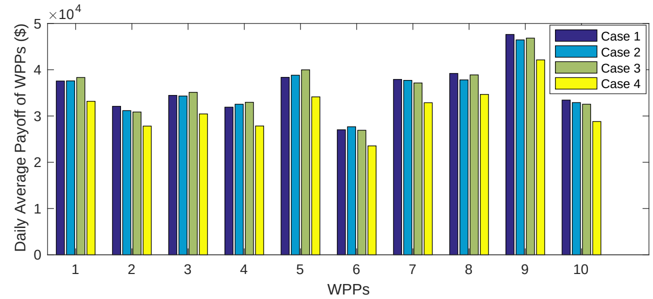

Fig. 1: Comparison of daily average payoffs of the WPPs.

Table 1: Total payoff of the aggregator

	Case 1	Case 2	Case 3	Case 4
Total Payoff ($)	10,428,257	10,352,581	10,428,257	9,148,024

The total payoffs of the aggregator for the four cases are shown in Table. 1. We note that the total payoffs in Case 1 and 3 are the same. This is expected because both PAMs achieve *social efficiency*, i.e., the total DA-commitments from the aggregator in both cases are the same optimal commitment $c_\mathcal{N}^\star$. The total payoff in Case 2 is slightly lower than the maximum achievable one in Case 1 and 3. This is due to the fact that $\sum_{i=1}^{N} c_i^{\star,sep} \neq c_\mathcal{N}^\star$.

6. CONCLUSION

We have studied the problem of payoff allocation for aggregating renewable power producers (RPPs). We have shown that a simple payoff allocation mechanism achieves efficiency, individual rationality, stability (in the core), and no collusion at the unique pure Nash Equilibrium (NE) of the non-cooperative game induced by the PAM among the RPPs. Moreover, we have provided a closed form expression of the unique pure NE.

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