Blind Estimation of States and Topology (BEST) in Power Systems

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***Abstract*—In this paper, we consider the problem of state estimation and topology identification in power systems. We assume the DC model of real power measurements with unknown voltage phases and an unknown admittance matrix. We show that this problem is equivalent to the blind source separation (BSS) problem, where the mixing matrix is a weighted Laplacian matrix. We propose two new Blind Estimation of States and Topology (BEST) methods for this problem. The first method, Cov-BEST, is based on utilizing the states' second-order statistics and the positive-definiteness of the reduced Laplacian matrix. The second method, Generalized Laplacian Separation (GLS)-BEST, is obtained by applying any general BSS method, followed by an approach that resolves the inherent BSS ambiguities by utilizing the Laplacian matrix properties. In contrast to existing methods, the proposed methods achieve full recovery of the topology matrix and are not limited to matrix eigenvectors estimation. The performance of the proposed methods is evaluated for a general network with an arbitrary number of buses and for the IEEE-14 bus system, and compared with the oracle state estimator.**

***Index Terms*—Blind source separation (BSS), Laplacian mixing matrix, Topology identification, State estimation, Power system monitoring**

I. Introduction

State estimation and network topology are critical components of modern Energy Management Systems (EMSs) for multiple monitoring purposes, including analysis, security, control, and stability assessment of power systems. In the DC model, the states are the bus voltage angles, while the grid topology includes the arrangement of loads or generators, transmission lines, transformers, and statuses of system devices. The grid topology is an integral part of state estimation and is essential for security, power market design, scheduling of connected devices, and optimization of electricity dispatchment. Usually, it is assumed that the EMS has precise knowledge of the grid topology [1]. However, this knowledge may not be available and may be incorrect due to malicious attacks [2], failure, opening and closing of switches, and the presence of new loads and generators. Thus, methods for state estimation and topology identification are crucial for obtaining an accurate system model and high power quality. Additionally, topology identification can be used for identifying faults and line outages, and for cybersecurity in the context of cyberattacks on the topology data.

Several approaches to topology identification have been proposed in the literature. Detecting topological changes has been studied in [3, 4] and the conditions for the detectability of topology errors are studied in [5]. Recently, a few papers have addressed blind estimation of the grid topology by observing multiple power injection measurements [6, 7], voltage and power data obtained by phasor measurement units (PMUs) [8], voltage measurements and their associated correlations [9-11], historical voltage phasor measurements and partially known grid topology [12], and electricity price based market data [13]. The methods proposed in [6, 7, 13] can reveal part of the grid topology, such as the grid connectivity and the eigenvectors of the topology matrix, but they cannot reconstruct the full topology matrix with exact scaling and true eigenvalues. In addition, these methods are highly dependent on the parameters used in the optimization process, on the initialization step, and on the sparsity level of the states [6] or of the topology matrix [7]. Blind source separation (BSS) [14-19] refers to the problem of recovering signals from several observed mixtures without prior knowledge of the sources and the mixing system. In the last decade, modern optimization and statistical methodologies have been shown to be powerful tools in power system problems (see e.g. [20-25]). In this context, applying BSS techniques for state and topology estimation seems a promising tool.

In this work, we consider the problem of topology identification in power systems with unknown states. We use the DC power-flow model with active power measurements. First, we show that this problem is equivalent to the problem of BSS with a weighted Laplacian mixing matrix, where the weights are determined by the branch susceptances. Then, we derive two Blind Estimation of States and Topology (BEST) methods: 1) Cov-BEST, which uses the states' second-order statistics (SOS) and the positive-definiteness of the Laplacian matrix; and 2) Generalized Laplacian Separation (GLS)-BEST, which is based on correcting any general BSS method by using the Laplacian matrix properties. To the best of our knowledge, these are the first published methods that provide full recovery of the topology, without any topology information and with unknown states. Finally, simulations demonstrate that the proposed methods are applicable for different network topologies.

In the rest of this paper vectors are denoted by boldface lowercase letters and matrices by boldface uppercase letters. The identity matrix of dimension $K \times K$ is denoted by $\mathbf{I}_K$, and $\mathbf{1}$ and $\mathbf{0}$ denote the constant one and zero vectors. The superscripts $(\cdot)^T$ and $(\cdot)^{-1}$ denote the transpose and matrix inverse operators, respectively. For a full-rank matrix $\mathbf{A}$, $\mathbf{A}^\dagger$ denotes the Moore-Penrose pseudo-inverse, $\mathbf{A}^\dagger \triangleq \left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{A}^T$. $\text{diag}(\mathbf{x})$ denotes the diagonal matrix with vector $\mathbf{x}$ on the diagonal. The mth element of the vector $\mathbf{a}$, the (m,q)th element of the matrix $\mathbf{A}$, and the $(m_1 : m_2 \times q_1 : q_2)$ submatrix of $\mathbf{A}$ are denoted by a_m, $[\mathbf{A}]_{m,q}$, and $[\mathbf{A}]_{m_1:m_2,q_1:q_2}$, respectively.

II. System Model

In this section, we present the power network as a graph, describe the DC model, and formulate the estimation problem.

A. Power systems as graphs

Power systems are characterized by buses that represent interconnections, generators or loads, denoted by the set $\mathcal{M} \stackrel{\triangle}{=} \{1,\ldots,M\}$, and a set of edges $\xi \stackrel{\triangle}{=} \{(m,k)\}$, where the edge (m,k) corresponds to the transmission line between buses m and k. Each line is characterized by the line admittance $Y_{m,k}$, $\forall (m,k) \in \xi$. Therefore, a power system can be represented as an undirected graph $\mathcal{G} = (\mathcal{M}, \xi)$, where $\mathcal{M}$ is the set of nodes (buses) and ξ is the set of edges (connected transmission lines). The cardinality $|\xi| = \frac{M(M-1)}{2}$ represents all possible connections in the system.

An arbitrary orientation is assigned to each edge $e_i = (m,k) \in \xi$, $i = 1,\ldots,\frac{M(M-1)}{2}$, where $e_1 = (1,2), e_2 = (1,3),\ldots,e_{\frac{M(M-1)}{2}} = (M, M-1)$. Then, the associated topology of the graph can be described by the oriented incidence matrix $\mathbf{B} \in \mathbb{R}^{M \times \frac{M(M-1)}{2}}$ [26], where the (m,i) element of $\mathbf{B}$ is given by

$$[\mathbf{B}]_{m,i} = \begin{cases} 1 & e_i = (m,k) \text{ is connected, } m = \text{source} \\ -1 & e_i = (m,k) \text{ is connected, } k = \text{source} \\ 0 & e_i = (m,k) \text{ is not connected} \end{cases}, \quad (1)$$

$\forall m = 1,\ldots,M$ and $i = 1,\ldots,\frac{M(M-1)}{2}$. Using the incidence matrix, the weighted Laplacian of the graph is defined as

$$\mathbf{H} \stackrel{\triangle}{=} \mathbf{B}\mathbf{Y}\mathbf{B}^T, \quad (2)$$

where $\mathbf{Y}$ is a diagonal weight matrix, in which $Y_{i,i} = Y_{m,k}$ contains the line admittance for the ith edge, $e_i = \{m,k\} \in \xi$, $i = 1,\ldots,\frac{M(M-1)}{2}$, i.e. $\forall m,k = 1,\ldots,M$, such that $m < k$. Note that the matrix $\mathbf{H}$ defined in (2) is a real, symmetric, and positive semidefinite matrix, which satisfies the null space property, $\mathbf{H}\mathbf{1} = \mathbf{0}$.

B. DC model and problem formulation

We consider the DC power flow model [1], which is based on the following assumptions on the network:

- Branches are considered lossless, which results in $Y_{m,k} = b_{m,k}$, where $b_{m,k}$ is the susceptance of the (m,k) branch.
- The bus voltage magnitudes, V_m, $m = 1,\ldots,M$, are approximated by 1 per unit (p.u.).
- Voltage angle differences across branches are small, such that $\sin(\theta_m - \theta_k) \approx \theta_m - \theta_k$, where θ_m, $m = 1,\ldots,M$, are the bus voltage angles.

Under these assumptions, the active power injected at bus m satisfies

$$p_m = -\sum_{k=1,k\neq m}^{M} b_{m,k} V_m V_k \sin(\theta_m - \theta_k)$$
$$\approx -\sum_{k=1,k\neq m}^{M} Y_{m,k}(\theta_m - \theta_k), \ \forall m = 1,\ldots,M. \quad (3)$$

Now, let $\mathbf{p}[n] \stackrel{\triangle}{=} [p_1[n],\ldots,p_M[n]]^T$ be the vector of active power injected and $\boldsymbol{\theta}[n] \stackrel{\triangle}{=} [\theta_1[n],\ldots,\theta_M[n]]^T$ the vector of voltage phase angles at time n, $\forall n = 0,\ldots,N-1$. Then, based on the model from (3), the linearized DC model of the network can be written as

$$\mathbf{p}[n] = \mathbf{H}\boldsymbol{\theta}[n], \quad n = 0,\ldots,N-1, \quad (4)$$

where the topology matrix $\mathbf{H}$ is defined in (2) and is considered static for a short-period of time and under normal operating conditions. It is assumed that $\mathbf{H}$ is an unknown deterministic matrix and $\boldsymbol{\theta}[n]$, $n = 0,\ldots,N-1$ are unknown random states. As is customary in the BSS framework (see e.g. [14-16]), the model in (4) is assumed to be noiseless. Notwithstanding, in the simulations, it is demonstrated that the developed methods are robust to the presence of noise.

Due to the null space property of the Laplacian matrix, $\mathbf{H}$, it has, at most, a rank of $M-1$ and the DC model from (4) is not invertible. Therefore, a reference bus is usually assumed to have zero voltage phase. Without loss of generality we assume in the following that $\theta_1[n] = 0$, $\forall n = 0,\ldots,N-1$. By imposing the constraint $\mathbf{H}\mathbf{1} = \mathbf{0}$ and the assumption regarding the reference bus, the model from (4) can be rewritten as

$$\mathbf{p}[n] = \mathbf{A}\tilde{\mathbf{H}}\tilde{\boldsymbol{\theta}}[n], \quad n = 0,\ldots,N-1, \quad (5)$$

where $\tilde{\mathbf{H}} \stackrel{\triangle}{=} \mathbf{H}_{2:M,2:M}$, $\tilde{\boldsymbol{\theta}}[n] \stackrel{\triangle}{=} [\theta_2[n],\theta_3[n],\ldots,\theta_M[n]]^T$, and $\mathbf{A} = \begin{bmatrix} -\mathbf{1}_{M-1}^T \\ \mathbf{I}_{M-1} \end{bmatrix} \in \mathbb{R}^{M\times(M-1)}$. The model in (5) is equivalent to

$$\tilde{\mathbf{p}}[n] = \tilde{\mathbf{H}}\tilde{\boldsymbol{\theta}}[n], \quad n = 0,\ldots,N-1, \quad (6)$$

where $\tilde{\mathbf{p}}[n] \stackrel{\triangle}{=} \mathbf{A}^\dagger \mathbf{p}[n]$ and $\tilde{\mathbf{H}}$ is called the 1st-reduced Laplacian (see e.g. p. 161 in [26] and [27]). Under the assumption that there are no unobservable islands in the grid, the reduced topology matrix, $\tilde{\mathbf{H}}$, is a full-$M-1$-rank matrix and, thus, can be identified. In addition, since $\mathbf{H}$ is a positive semidefinte Laplacian matrix, then, $\tilde{\mathbf{H}}$ is also a symmetric, positive semidefinte matrix with nonpositive off-diagonal elements and is a diagonally dominant matrix, i.e. $0 \leq \sum_{m=1}^{M-1}[\tilde{\mathbf{H}}]_{k,m}$, $\forall k = 1,\ldots,M-1$.

III. System identification and BSS

In this section, we develop two BEST-type methods for joint reconstruction of the matrix $\tilde{\mathbf{H}}$ and the states $\tilde{\boldsymbol{\theta}}[n]$, $n = 0,\ldots,N-1$. This problem can be interpreted as a BSS problem with a reduced Laplacian mixing matrix, which satisfies the aforementioned properties. The original topology matrix, $\mathbf{H}$, can be reconstructed from $\tilde{\mathbf{H}}$ by using the relation $\mathbf{H} = \mathbf{A}\tilde{\mathbf{H}}\mathbf{A}^T$. In addition, the states $\tilde{\boldsymbol{\theta}}[n], n = 0,\ldots,N-1$, can be estimated under the two methods, based on the estimated topology, $\hat{\tilde{\mathbf{H}}}$, and the measurements, $\tilde{\mathbf{p}}[n], n = 0,\ldots,N-1$, by

$$\hat{\tilde{\boldsymbol{\theta}}}[n] = \hat{\tilde{\mathbf{H}}}^{-1}\tilde{\mathbf{p}}[n], \quad n = 0,\ldots,N-1. \quad (7)$$

A. Cov-BEST

The proposed Cov-BEST method is based on the states SOS. For simplicity it is developed under the assumption that the state sequence $\tilde{\boldsymbol{\theta}}[n]$, $n = 0,\ldots,N-1$, is a stationary Gaussian time-independent random process, $\tilde{\boldsymbol{\theta}}[n] \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_{\tilde{\boldsymbol{\theta}}})$, but it can be used for any zero-mean time-independent process with a known covariance matrix. This SOS can be obtained, for example, by using historical statistical data. Under these assumptions and by using the model in (6) the normalized log likelihood of $\tilde{\mathbf{p}}[n]$, $n=0,\ldots,N-1$ is given by [15]:

$$\psi(\tilde{\mathbf{H}}) = const - \frac{1}{2}\text{trace}\left(\hat{\boldsymbol{\Sigma}}_{\tilde{\mathbf{p}}}\left(\tilde{\mathbf{H}}^T\boldsymbol{\Sigma}_{\tilde{\boldsymbol{\theta}}}\tilde{\mathbf{H}}\right)^{-1}\right) - \log|\tilde{\mathbf{H}}|, \quad (8)$$

where $const$ is independent of $\tilde{\mathbf{H}}$, $\hat{\boldsymbol{\Sigma}}_{\tilde{\mathbf{p}}} \triangleq \frac{1}{N}\sum_{n=0}^{N-1}\tilde{\mathbf{p}}[n]\tilde{\mathbf{p}}^T[n]$, and it is assumed that the matrices $\boldsymbol{\Sigma}_{\tilde{\boldsymbol{\theta}}}$ and $\tilde{\mathbf{H}}$ are non-singular matrices. The derivative of (8) w.r.t. $\tilde{\mathbf{H}}$ is given by

$$\frac{\partial\psi(\tilde{\mathbf{H}})}{\partial\tilde{\mathbf{H}}} = \tilde{\mathbf{H}}^{-T}\hat{\boldsymbol{\Sigma}}_{\tilde{\mathbf{p}}}\tilde{\mathbf{H}}^{-1}\boldsymbol{\Sigma}_{\tilde{\boldsymbol{\theta}}}^{-1}\tilde{\mathbf{H}}^{-T} - \tilde{\mathbf{H}}^{-T}. \quad (9)$$

By equating (9) to zero and substituting the symmetry $\tilde{\mathbf{H}}^T = \tilde{\mathbf{H}}$, one obtains that the maximum likelihood (ML) estimator for a symmetric positive definite mixing matrix satisfies

$$\hat{\tilde{\mathbf{H}}} = \boldsymbol{\Sigma}_{\tilde{\boldsymbol{\theta}}}^{-1}\hat{\tilde{\mathbf{H}}}^{-1}\hat{\boldsymbol{\Sigma}}_{\tilde{\mathbf{p}}}. \quad (10)$$

It can be verified that

$$\hat{\tilde{\mathbf{H}}} = \boldsymbol{\Sigma}_{\tilde{\boldsymbol{\theta}}}^{-\frac{1}{2}}(\boldsymbol{\Sigma}_{\tilde{\boldsymbol{\theta}}}^{\frac{1}{2}}\hat{\boldsymbol{\Sigma}}_{\tilde{\mathbf{p}}}\boldsymbol{\Sigma}_{\tilde{\boldsymbol{\theta}}}^{\frac{1}{2}})^{\frac{1}{2}}\boldsymbol{\Sigma}_{\tilde{\boldsymbol{\theta}}}^{-\frac{1}{2}} \quad (11)$$

is the unique solution of (10), where we use the decompositions $\boldsymbol{\Sigma}_{\tilde{\theta}} = \boldsymbol{\Sigma}_{\tilde{\theta}}^{\frac{1}{2}}\boldsymbol{\Sigma}_{\tilde{\theta}}^{\frac{1}{2}}$ and $\boldsymbol{\Sigma}_{\tilde{\theta}}^{-1} = \boldsymbol{\Sigma}_{\tilde{\theta}}^{-\frac{1}{2}}\boldsymbol{\Sigma}_{\tilde{\theta}}^{-\frac{1}{2}}$, In addition, if the state expectation is nonzero and unknown, then $\hat{\boldsymbol{\Sigma}}_{\mathbf{P}}$ can be replaced by the sample covariance matrix [28]

$$\hat{\boldsymbol{\Sigma}}_{\tilde{\mathbf{p}}} = \frac{1}{N-1}\sum_{n=0}^{N-1}\left(\tilde{\mathbf{p}}[n]-\bar{\mathbf{p}}\right)\left(\tilde{\mathbf{p}}[n]-\bar{\mathbf{p}}\right)^T, \quad (12)$$

where $\bar{\mathbf{p}} \triangleq \frac{1}{N}\sum_{n=0}^{N-1}\tilde{\mathbf{p}}[n]$ is the sample mean. In future research, the ML process should be extended by taking into account additional constraints of the reduced Laplacian matrix. The Cov-BEST algorithm is summarized in Algorithm 1.

Algorithm 1: Cov-BEST Algorithm

Input: • Observations $\mathbf{p}[n]$, $n=0,\ldots,N-1$.
• State covariance matrix, $\boldsymbol{\Sigma}_{\tilde{\theta}}$.

Output: Estimators $\hat{\mathbf{H}}$ and $\hat{\boldsymbol{\theta}}[n]$, $n=0,\ldots,N-1$.

Algorithm Steps:

1) Compute the modified input $\tilde{\mathbf{p}}[n] = \mathbf{A}^\dagger\mathbf{p}[n]$.
2) Compute the sample covariance estimator, $\hat{\boldsymbol{\Sigma}}_{\tilde{\mathbf{p}}}$, from (12) by using $\tilde{\mathbf{p}}[n]$, $n=0,\ldots,N-1$.
3) Evaluate $\hat{\tilde{\mathbf{H}}} = \boldsymbol{\Sigma}_{\tilde{\theta}}^{-\frac{1}{2}}(\boldsymbol{\Sigma}_{\tilde{\theta}}^{\frac{1}{2}}\hat{\boldsymbol{\Sigma}}_{\tilde{\mathbf{p}}}\boldsymbol{\Sigma}_{\tilde{\theta}}^{\frac{1}{2}})^{\frac{1}{2}}\boldsymbol{\Sigma}_{\tilde{\theta}}^{-\frac{1}{2}}$.
4) Estimate the sources by (7): $\hat{\tilde{\boldsymbol{\theta}}}[n] = \hat{\tilde{\mathbf{H}}}^{-1}\tilde{\mathbf{p}}[n]$ and, with the reference bus, $\hat{\boldsymbol{\theta}}[n] = [0, \hat{\tilde{\boldsymbol{\theta}}}^T[n]]^T$.
5) Evaluate the full topology matrix $\hat{\mathbf{H}} = \mathbf{A}\hat{\tilde{\mathbf{H}}}\mathbf{A}^T$.

B. GLS-BEST

The GLS-BEST is a two-stage method which assumes that states have an unknown non-Gaussian distribution with known distribution's support (range of values). The first stage of the GLS-BEST method is based on applying any general BSS method [14-18] on $\tilde{\mathbf{p}}[n]$, $n=0,\ldots,N-1$, from (6) to obtain the initial estimate of the mixing matrix, $\hat{\tilde{\mathbf{H}}}^{(0)}$. The second stage is based on correcting $\hat{\tilde{\mathbf{H}}}^{(0)}$ by using the characteristics of the reduced Laplacian matrix and knowledge of the range of values of the states to resolve the inherent scale and permutation ambiguities of BSS methods [14, 15]. In particular, the correction of the estimated topology is performed as follows:

1) Permutation - As described in Section II, $\tilde{\mathbf{H}}$ satisfies $\max_{m=1,\ldots,M-1}|[\tilde{\mathbf{H}}]_{k,m}| = |[\tilde{\mathbf{H}}]_{k,k}|$. Therefore, in order to solve the permutation ambiguity, the columns of $\hat{\tilde{\mathbf{H}}}^{(0)}$ are reordered such that the element with the maximum absolute value in each row is placed on the diagonal of the modified matrix. The reordering is performed from the first row to the $M-1$ row, where a column that was once reordered will not be reordered again. The reordered matrix is denoted by $\hat{\tilde{\mathbf{H}}}^{(1)}$.
2) Sign - To force the diagonal values of $\hat{\tilde{\mathbf{H}}}^{(1)}$ to be non negative, as required for a reduced Laplacian matrix (see Section II), we multiply each column of $\hat{\tilde{\mathbf{H}}}^{(1)}$ by ± 1 to obtain $\hat{\tilde{\mathbf{H}}}^{(2)}$ with a nonnegative diagonal.
3) Scale - In order to solve the scale ambiguity of $\hat{\tilde{\mathbf{H}}}^{(2)}$, we use the small angles assumption of the DC model and assume that $|\theta_m[n]| \leq T_m$, $m=2,\ldots,M$, $n=0,\ldots,N-1$, where $\{T_m\}_{m=2}^M$ are known thresholds. Thus, we set the final topology estimator to $\hat{\tilde{\mathbf{H}}} = \hat{\tilde{\mathbf{H}}}^{(2)}\text{diag}(f_2,\ldots,f_M)$, where
$$f_m = \frac{1}{T_m}\max_{n=0,\ldots,N-1}|\hat{\tilde{\theta}}_m^{(2)}[n]|,\ m=2,\ldots,M-1 \quad (13)$$
and $\hat{\tilde{\boldsymbol{\theta}}}^{(2)} \triangleq (\hat{\tilde{\mathbf{H}}}^{(2)})^{-1}\tilde{\mathbf{p}}[n]$, $n=0,\ldots,N-1$.

It should be noted that for this algorithm there is no unique solution, as, for example, for different reordering methods at the permutation stage there can be different solutions. The algorithm is summarized in Algorithm 2.

Algorithm 2: GLS-BEST

Input: • Observations $\mathbf{p}[n]$, $n=0,\ldots,N-1$.
• Thresholds of the voltage angles, $\{T_m\}_{m=2}^M$.

Output: Estimators $\hat{\mathbf{H}}$ and $\hat{\boldsymbol{\theta}}[n]$, $n=0,\ldots,N-1$.

Algorithm Steps:

1) Compute the modified input $\tilde{\mathbf{p}}[n] = \mathbf{A}^\dagger\mathbf{p}[n]$.
2) Apply any BSS algorithm on $\tilde{\mathbf{p}}[n]$, $n=0,\ldots,N-1$, to obtain the mixing matrix $\hat{\tilde{\mathbf{H}}}^{(0)}$.
3) Reorder the columns of $\hat{\tilde{\mathbf{H}}}^{(0)}$ to obtain $\hat{\tilde{\mathbf{H}}}^{(1)}$, such that the maximal absolute value of the mth row of the matrix appears at the mth column, $m=1,\ldots,M-1$.
4) If $[\hat{\tilde{\mathbf{H}}}^{(1)}]_{m,m}$ is negative, multiply the mth column of $\hat{\tilde{\mathbf{H}}}^{(1)}$ by -1 to obtain $\hat{\tilde{\mathbf{H}}}^{(2)}$, $\forall m=1,\ldots,M-1$.
5) Compute f_m, $m=2,\ldots,M$ from (13).
6) Evaluate $\hat{\tilde{\mathbf{H}}} = \hat{\tilde{\mathbf{H}}}^{(2)}\text{diag}(f_2,\ldots,f_M)$.
7) Estimate the sources by (7): $\hat{\tilde{\boldsymbol{\theta}}}[n] = \hat{\tilde{\mathbf{H}}}^{-1}\tilde{\mathbf{p}}[n]$ and, with the reference bus, $\hat{\boldsymbol{\theta}}[n] = [0, \hat{\tilde{\boldsymbol{\theta}}}^T[n]]^T$.
8) Evaluate $\hat{\mathbf{H}} = \mathbf{A}\hat{\tilde{\mathbf{H}}}\mathbf{A}^T$.

C. Identifiability

It is proved in [6] that, based on the DC model in (5), the Laplacian matrix, $\mathbf{H}$, cannot be uniquely identified using solely power injection data. In this paper, we impose further assumptions on the system states, which enables full topology matrix recovery. In particular, the Cov-BEST method utilizes the states' SOS, while the GLS-BEST method uses the value range of the states. It should be noted that the support of $\mathbf{H}$, i.e. the network connectivity, can be recovered using the GLS-BEST method without knowledge of the value ranges, by simply applying a threshold operator on $\hat{\tilde{\mathbf{H}}}^{(2)}$.

IV. Simulations

In this section, we evaluate the performance of the two proposed BEST algorithms by using 200 Monte-Carlo simulations. The power measurements are generated using (4), with additional zero-mean Gaussian noise with variance σ^2. It is assumed that the states and noise sequences are mutually independent with a diagonal covariance matrix. The first stage of the GLS-BEST method is implemented by the FastICA algorithm [29], which requires a large number of measurements and high SNR conditions. Thus, the performance of the GLS-BEST method is presented for a limited range of SNR values and numbers of buses.

A. Case study: 14-bus system

In this subsection, we implement the proposed methods for the IEEE 14-bus system, where the branch susceptances for each line are taken from [30]. The states, $\boldsymbol{\theta}[n]$, $n = 0, \ldots, N-1$, are modeled as uniformly distributed measurements around the nominal value of the buses, $\boldsymbol{\theta}^{(\text{nom})}$, according to [30]. In order to maintain the DC model assumption of small angles, we set $\boldsymbol{\theta}[n] \in \left[\boldsymbol{\theta}^{(\text{nom})} - (\frac{\pi}{8} - |\boldsymbol{\theta}^{(\text{nom})}|), \boldsymbol{\theta}^{(\text{nom})} + (\frac{\pi}{8} - |\boldsymbol{\theta}^{(\text{nom})}|)\right]$, $n = 0, \ldots, N-1$. We also implemented the oracle state estimator, which assumes perfect knowledge of the topology matrix, $\mathbf{H}$, and is given by

$$\hat{\tilde{\boldsymbol{\theta}}}^{(\text{oracle})}[n] = \tilde{\mathbf{H}}^{-1}\tilde{\mathbf{p}}[n], \quad n = 0, \ldots, N-1. \tag{14}$$

The oracle estimation was implemented with $N = 50$ measurements, since the average MSE over $n = 0, \ldots, N-1$, is independent of N for a known topology.

In Fig. 1, the mean-squared-error (MSE) of the state estimators is presented versus SNR, where the SNR is defined as $\text{SNR} = \frac{1}{\sigma^2 M}\text{trace}(\mathbf{H}\boldsymbol{\Sigma}_{\tilde{\theta}}\mathbf{H})$. The performance of the Cov-BEST and the GLS-BEST methods is presented for $N = 50, 1000$ and for $N = 1000, 10000$, respectively. It can be seen that the MSE of the state estimation by the Cov-BEST method is lower than the MSE of the GLS-BEST estimator. Additionally, for high SNRs, the state estimation performance of Cov-BEST with estimated topology converges to that of the oracle method, which uses the true topology. Therefore, we can conclude that for high SNRs the topology estimation convergences to the true topology.

B. Case study: random topology matrices

In order to conduct an experiment with varying numbers of buses, $\mathbf{H}$ was generated by taking off-diagonal elements with a uniform distribution satisfying $[\mathbf{H}]_{m,k} \in [-1, 0]$. The state vectors, $\boldsymbol{\theta}[n]$, were generated with a uniform zero-mean distribution, satisfying $\theta_m[n] \in [-\frac{\pi}{8}, \frac{\pi}{8}]$, $n = 0, \ldots, N-1$, $m = 2, \ldots, M$. Fig. 2 shows the root normalized MSE

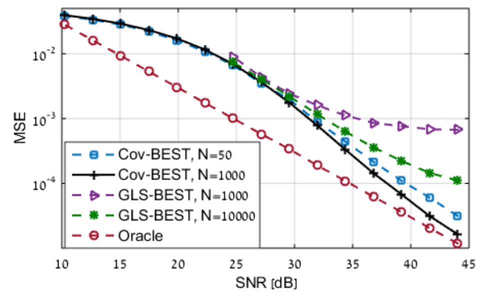

Fig. 1. MSE of state estimators by Cov-BEST, GLS-BEST, and oracle methods for IEEE-14 bus system.

(RNMSE) of the topology estimator, $\frac{||\mathbf{H}-\hat{\mathbf{H}}||_F}{||\mathbf{H}||_F}$, in which $||\cdot||_F$ denotes the Frobenius norm, versus the number of buses in the system for SNR$= 30$dB. It can be seen that the performance of the Cov-BEST is significantly better than those of the GLS-BEST and that as the number of buses increases, the RNMSE increases and more power measurements need to be taken to keep the same level of accuracy as for small topologies.

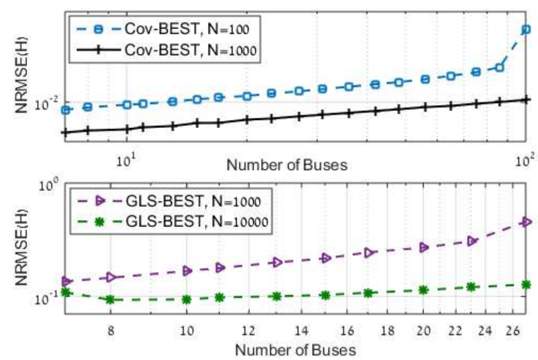

Fig. 2. RNMSE of the grid topology estimation by by Cov-BEST and GLS-BEST methods versus number of buses.

V. Conclusion

In this paper we introduce two novel methods for blind estimation of states and topology in power systems, given active power measurements. The first method, Cov-BEST, is based on using the states' SOS and the positive-definiteness of the reduced Laplacian matrix. The second method, GLS-BEST, is a two-stage method that performs a conventional BSS on the power measurements and then, resolves the inherent permutation and scaling ambiguities by using the unique properties of the Laplacian topology matrix and knowledge of the value range of the states. The proposed methods are non-iterative methods and, thus, do not suffer from problems of convergence and initial guess. Simulations show that the proposed methods are feasible and succeed in reconstructing the topology and estimating the states, and that the state estimator by the Cov-BEST converges to the oracle state estimator, which assumes perfect knowledge of the topology. Topics for future research include incorporating sparsity constraints and assuming a noisy measurement model.

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