Operational Semantics

Motivation

- Expressing the meaning of a programming language in natural language is error prone —— ambiguous
- Formal semantics gives an unambiguous definition of what a program written in the language should do —— unambiguous
  - ✓Understand the subtleties of the language
  - ✓Offer a formal reference and a correctness definition for implementers of tools (parsers, compilers, interpreters, debuggers, etc)
  - ✓Prove global properties of any program written in the language, e.g., assertion
  - ✓Verify programs against formal specifications
  - ✓Prove two different programs are equivalent/non-equivalent
  - ✓Form a computer readable version of the semantics, an interpreter can be automatically generated (full compiler generation is not yet feasible), like K Framework

Formal semantics

Operational semantics:

- The meaning of a construct is specified by the computation it induces when it is executed on a machine. In particular, it is of interest how the effect of a computation is produced.

Denotational semantics:

- Meanings are modelled by mathematical objects that represent the effect of executing the constructs. Thus only the effect is of interest, not how it is obtained.

Axiomatic semantics:

- Specific properties of the effect of executing the constructs are expressed as assertions. Thus there may be aspects of the executions that are ignored.

Operational Semantics

- Operational semantics defines program executions:
  - ✓Sequence of steps, formulated as transitions of an abstract machine/interpreter

- Configurations of the abstract machine include:
  - ✓Expression/statements being evaluated/executed
  - ✓States: abstract description of registers, memory and other data structures involved in computation

- Most useful for specifying implementations
- This is what we will use for Cool

Operational Semantics

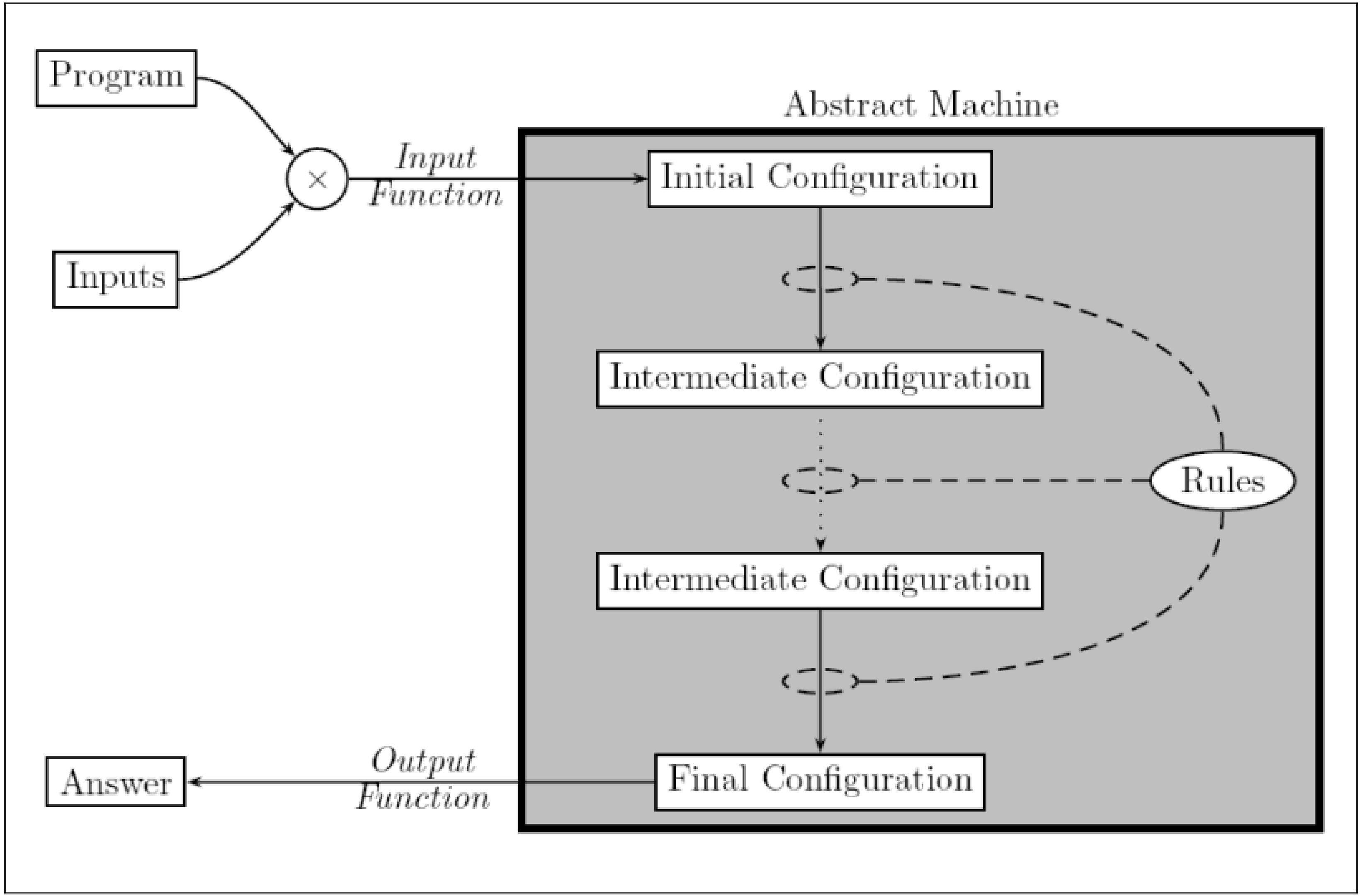


Figure taken from Franklyn Turbak and David Gifford's Design Concepts in Programming Languages.

Other Kinds of Semantics

Denotational semantics

- The meaning of a program is expressed as a mathematical object/denotation
- Very elegant but quite complicated

  e.g., Functional languages often translate the language into domain theory

Axiomatic semantics

- The meaning of a program is described the logical axioms, e.g., Hoare logic
- Useful for checking that programs satisfy certain correctness properties using proof systems, e.g., that the quick sort function sorts an array
- The foundation of many program verification systems

Operational semantics

z:=x; x:=y; y:=z

Configuration: Variable-> Value, e.g., [x→5, y→7, z→0]

z:=x; x:=y; y:=z, [x→5, y→7, z→0]
⇒ x:=y; y:=z, [x→5, y→7, z→5]
⇒ y:=z, [x→7, y→7, z→5]
⇒ [x→7, y→5, z→5]

- This explanation gives an abstraction of how the program is executed on a machine.
- It is important to observe that it is indeed an abstraction
- We ignore details such as the use of registers and addresses for variables.
- So the operational semantics is rather independent of machine architectures and implementation strategies.

Denotational semantics

z:=x; x:=y; y:=z

Mathematical object: a function **F**: $2^{\text{Configuration}}$->$2^{\text{Configuration}}$

F[z:=x] = λ c: c[z→c(x)]   **F**[x:=y] = λ c: c[x→c(y)]   **F**[y:=z] = λ c: c[y→c(z)]

F[z:=x; x:=y; y:=z] = **F**[y:=z]∘ **F**[x:=y] ∘ **F**[z:=x]

F[z:=x; x:=y; y:=z] ([x→5, y→7, z→0])

= (**F**[y:=z]∘ **F**[x:=y] ∘ **F**[z:=x]) ([x→5, y→7, z→0])

= **F**[y:=z] (**F**[x:=y] (**F**[z:=x] ([x→5, y→7, z→0])))

= **F**[y:=z] (**F**[x:=y] ([x→5, y→7, z→5]))

= **F**[y:=z] ([x→7, y→7, z→5])

= [x→7, y→5, z→5]

- The benefits: abstracts away from how programs are executed.
- Amounts to reasoning about mathematical objects.
- But, have to stablish a firm mathematical basis for denotational semantics, and this task turns out not to be entirely trivial

Axiomatic semantics

$z:=x;\ x:=y;\ y:=z$

{Precondition} P {Postcondition}

$\{x=n \land y=m\}\ z:=x\ \{z=n \land y=m\}$

$\{z=n \land y=m\}\ x:=y\ \{z=n \land x=m\}$

$\{z=n \land x=m\}\ y:=z\ \{y=n \land x=m\}$

$\{x=n \land y=m\}\ z:=x;\ x:=y\ \{z=n \land x=m\}$

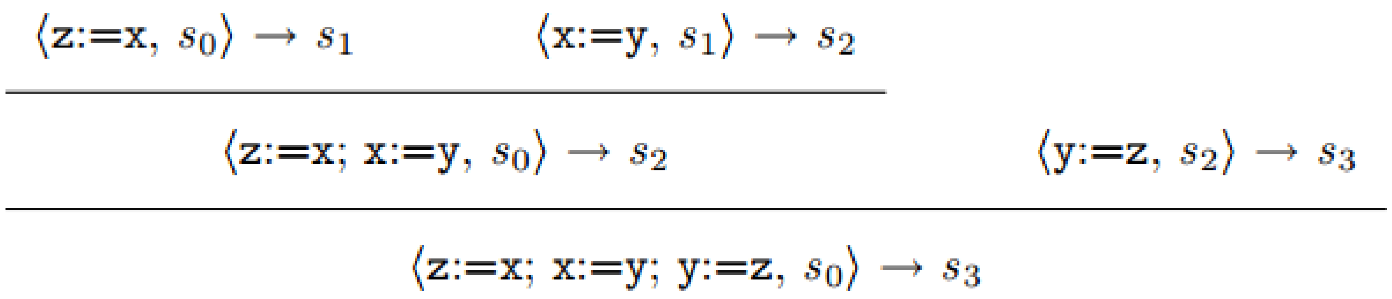
$\{x=n \land y=m\}\ z:=x;\ x:=y;\ y:=z\ \{y=n \land x=m\}$

- The axiomatic semantics provides a logical system for proving partial correctness properties of individual programs.
- Partial correctness: A program is partially correct, with respect to a precondition and a postcondition, if whenever the initial state fulfils the precondition and the program terminates, then the final state is guaranteed to fulfil the postcondition
- Total correctness: partial correctness + termination

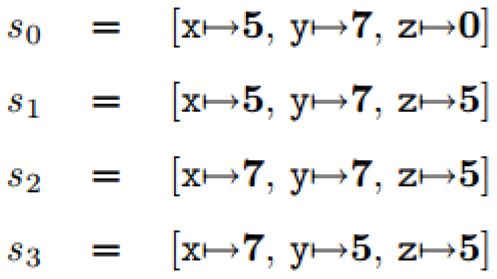
Operational Semantics

- Small step semantics (structural operational semantics, SOS)
- Big step semantics (natural semantics)
  - ✓differs from SOS by hiding even more execution details.

Natural semantics is represented by the derivation tree



$$\frac{\dfrac{\langle \mathrm{z{:=}x},\ s_0\rangle \to s_1 \qquad \langle \mathrm{x{:=}y},\ s_1\rangle \to s_2}{\langle \mathrm{z{:=}x;\ x{:=}y},\ s_0\rangle \to s_2} \qquad \langle \mathrm{y{:=}z},\ s_2\rangle \to s_3}{\langle \mathrm{z{:=}x;\ x{:=}y;\ y{:=}z},\ s_0\rangle \to s_3}$$



$$\begin{aligned}
s_0 &= [\mathrm{x}\mapsto\mathbf{5},\ \mathrm{y}\mapsto\mathbf{7},\ \mathrm{z}\mapsto\mathbf{0}]\\
s_1 &= [\mathrm{x}\mapsto\mathbf{5},\ \mathrm{y}\mapsto\mathbf{7},\ \mathrm{z}\mapsto\mathbf{5}]\\
s_2 &= [\mathrm{x}\mapsto\mathbf{7},\ \mathrm{y}\mapsto\mathbf{7},\ \mathrm{z}\mapsto\mathbf{5}]\\
s_3 &= [\mathrm{x}\mapsto\mathbf{7},\ \mathrm{y}\mapsto\mathbf{5},\ \mathrm{z}\mapsto\mathbf{5}]
\end{aligned}$$

<z:=x; x:=y; y:=z, s_0> →s_3

SOS

Configuration: a function Variable-> Value

- z:=x; x:=y; y:=z, [x→5, y→7, z→0]
- ⇒ x:=y; y:=z, [x→5, y→7, z→5]
- ⇒ y:=z, [x→7, y→7, z→5]
- ⇒ [x→7, y→5, z→5]

hidden the explanation above of how it was actually obtained

Operational Semantics for COOL

- Once, again we introduce a formal notation
  - Using logical rules of inference, just like for typing

O, M, C |- **e : T**

  - Under Context (O, M, C), e has type T

- We try something similar for evaluation

Context |- e : v

  - Under Context, e evaluates to the value v

Example of Inference Rule for Operational Semantics

$$\frac{\begin{array}{c}\textbf{Context |- } e_1 : 2 \\ \textbf{Context |- } e_2 : 3\end{array}}{\textbf{Context |- } e_1 + e_2 : 5}$$

What Contexts Are Needed?

- In general the result of evaluating an expression depends on the result of evaluating its subexpressions

- The logical rules specify everything that is needed to evaluate an expression

Contexts

Contexts are needed to handle variables

x = 1; y = x + 2; x = 3

- We need to keep track of values of variables
- We need to allow variables to change their values during the evaluation

We track variables and their values with:

- An environment E: tells us at what address in memory is the value of a variable stored
- A store S: tells us what is the contents of a memory location

Variable Environments

- A variable environment E is a map from variable names to locations
- Tells in what memory location the value of a variable is stored
- Keeps track of which variables are in scope
- Example:

$$E = [x : l_1, y : l_2]$$

- To lookup a variable a in environment E we write E(x)

Stores

- A store S maps memory locations to values

  Example:

  $$S = [l_1 \rightarrow 2, l_2 \rightarrow 3]$$

- To lookup the contents of a location l_1 in store S we write $S(l_1)$
- To perform an assignment of 5 to location l_1, we write $S[5/l_1]$
  - This denotes a new store S' such that

    $$S'(l_1) = 5$$

    $$S'(l) = S(l) \text{ if } l \neq l_1$$

Cool Values

- All values in Cool are objects
  - ✓All objects are instances of some class (the dynamic type of the object)
- To denote a Cool object we use the notation

$$v = X(a_1 = l_1, \ldots, a_n = l_n)$$

where

- ✓X is the dynamic type of the object
- ✓a_i are the attributes (including those inherited)
- ✓l_i are the locations where the values of attributes are stored
- ✓The value v is a member of class X containing the attributes $a_1,\ldots,a_n$ whose locations are $l_1,\ldots,l_n$.

Cool Values (Cont.)

- Special cases (classes without attributes)
  - ✓Int(5) the integer 5
  - ✓Bool(true) the boolean true
  - ✓String(4, “Cool”) the string “Cool” of length 4
- There is a special value void that is a member of all types
  - ✓No operations can be performed on it
  - ✓Except for the test isvoid
  - ✓Concrete implementations might use NULL here

Operational Rules of Cool

- The evaluation judgment is

so, E, S |- e : v, S’

read:

✓Given so the current value of the self object

✓E the current variable environment

✓S the current store

✓If the evaluation of e terminates then e evaluates to v, and resulting the new store is S’

Notes

- The “result” of evaluating an expression is a value and a new store
- Changes to the store model the side-effects
- The variable environment does not change, nor does the value of self
- self is just the object to which the identifier self refers if self appears in the expression.
- We do not place self in the environment and store?
- Because self is not a variable—it cannot be assigned to
- The operational semantics allows for nonterminating evaluations
- We define one rule for each kind of expression

Operational Semantics for Base Values

$$\frac{}{\text{so, E, S } \mathbf{|\text{-}} \text{ true : Bool(true), S}}$$

$$\frac{}{\text{so, E, S } \mathbf{|\text{-}} \text{ false: Bool(false), S}}$$

$$\frac{\text{2 is an integer literal}}{\text{so, E, S } \mathbf{|\text{-}} \text{ 2: Int(2), S}}$$

$$\frac{\text{“abc” is a string literal} \quad \text{3 is the length of s}}{\text{so, E, S } \mathbf{|\text{-}} \text{ s: String(3,”abc”), S}}$$

No side effects in these cases

-(the store does not change)

Operational Semantics of Variable References

$$\frac{\begin{array}{c} E(x) = l_x \\ S(l_x) = v \end{array}}{so, E, S \vdash x: v, S}$$

Note the double lookup of variables

- First from name to location
- Then from location to value

The store does not change

A special case:

$$\frac{}{so, E, S \vdash self: so, S}$$

Operational Semantics of Assignment

$$\frac{\begin{array}{c} so, E, S \vdash e: v, S_1 \\ E(x) = l_x \\ S_2 = S_1[v/l_x] \end{array}}{so, E, S \vdash x \leftarrow e: v, S_2}$$

A three step process

- Evaluate the right hand side e
  - ⇒ a value v and a new store S_1
- Fetch the location l_x of the assigned variable x
- The result is the value v and an updated store S_2

Operational Semantics of Conditionals

$$\frac{\begin{array}{c}\text{so, E, S} \mid\!- e_1 : \text{Bool(true)}, S_1 \\ \text{so, E, } S_1 \mid\!- e_2 : \text{v}, S_2\end{array}}{\text{so, E, S} \mid\!- \text{if } e_1 \text{ then } e_2 \text{ else } e_3 : \text{v}, S_2}$$

$$\frac{\begin{array}{c}\text{so, E, S} \mid\!- e_1 : \text{Bool(false)}, S_1 \\ \text{so, E, } S_1 \mid\!- e_3 : \text{v}, S_2\end{array}}{\text{so, E, S} \mid\!- \text{if } e_1 \text{ then } e_2 \text{ else } e_3 : \text{v}, S_2}$$

- The “threading” of the store enforces an evaluation sequence
  - e_1 must be evaluated first to produce S_1
  - Then e_2 or e_3 can be evaluated
- The result of evaluating e_1 is a boolean object
  - The typing rules ensure this

Operational Semantics of Sequences

$$\frac{\begin{array}{c} so, E, S \vdash e_1 : v_1, S_1 \\ so, E, S_1 \vdash e_2 : v_2, S_2 \end{array}}{so, E, S \vdash e_1 ; e_2 : v_2, S_2}$$

- Only the last value is used
- But all the side-effects are collected in stores

Operational Semantics of while (I)

$$\frac{\text{so, E, S} \;\vert\!\text{-}\; e_1 : \text{Bool(false)}, S_1}{\text{so, E, S} \;\vert\!\text{-}\; \text{while } e_1 \text{ loop } e_2 \text{ pool : void}, S_1}$$

$$\frac{\begin{array}{c}\text{so, E, S} \;\vert\!\text{-}\; e_1 : \text{Bool(true)}, S_1 \\ \text{so, E, } S_1 \;\vert\!\text{-}\; e_2 : v, S_2 \\ \text{so, E, } S_2 \;\vert\!\text{-}\; \text{while } e_1 \text{ loop } e_2 \text{ pool : void}, S_3\end{array}}{\text{so, E, S} \;\vert\!\text{-}\; \text{while } e_1 \text{ loop } e_2 \text{ pool : void}, S_3}$$

- If e_1 evaluates to Bool(false) then the loop terminates immediately
  - With the side-effects from the evaluation of e_1
  - And with result value void
  - The typing rules ensure that e_1 evaluates to a boolean object
- Otherwise
  - Note the sequencing ($S \rightarrow S_1 \rightarrow S_2 \rightarrow S_3$)
  - Note how looping is expressed

    Evaluation of “while …” is expressed in terms
    of the evaluation of itself in another state
  - The result v of e_2 is discarded, only the side-effect is preserved

Operational Semantics of let Expressions (I)

$$\frac{\begin{array}{c} \text{so}, E, S \vdash e_1 : v_1, S_1 \\ \text{so}, ?, ? \vdash e_2 : v_2, S_2 \end{array}}{\text{so}, E, S \vdash \text{let } x \text{: } T \leftarrow e_1 \text{ in } e_2 : v_2, S_2}$$

$$\frac{\begin{array}{c} \text{so}, E, S \vdash e_1 : v_1, S_1 \\ l_{new} = \text{newloc}(S_1) \\ \text{so}, E[l_{new}/x], S_1[v_1/l_{new}] \vdash e_2 : v_2, S_2 \end{array}}{\text{so}, E, S \vdash \text{let } x : T \leftarrow e_1 \text{ in } e_2 : v_2, S_2}$$

- What is the context in which e_2 must be evaluated?
  - Environment like E but with a new binding of x to a fresh location l_{new}
  - Store like S_1 but with l_{new} mapped to v_1

- lnew = newloc(S): lnew is a location that is not already used in S
  - Think of newloc as the dynamic memory allocation function

Default Values

For each class A there is a default value denoted by D_A

- D_{int} = Int(0)
- D_{bool} = Bool(false)
- D_{string} = String(0, “”)
- D_A = void (for another class A)

For a class A we write

class(A) = (a_1:T_1←e_1, …,a_n:T_n←e_n) // class mapping

where

- a_i are the attributes (including the inherited ones)
- T_i are their declared types
- e_i are the initializers

Operational Semantics of new

- Consider the expression new T
- Informal semantics
  - ✓ Allocate new locations to hold the values for all attributes of an object of class T

    Essentially, allocate a new object
  - ✓ Initialize those locations with the default values of attributes
  - ✓ Evaluate the initializers and set the resulting attribute values
  - ✓ Return the newly allocated object
- Observation: new SELF_TYPE allocates an object with the same dynamic type as self

Operational Semantics of new

allocate the object

initialize it

$$\frac{\begin{array}{c} T_0 = \text{if } T == \text{SELF\_TYPE and } so = X(\ldots) \text{ then } X \text{ else } T \\ \text{class}(T_0) = (a_1 : T_1 \leftarrow e_1, \ldots, a_n : T_n \leftarrow e_n) \\ l_i = \text{newloc}(S) \text{ for } i = 1,\ldots,n \\ v = T_0(a_1 = l_1, \ldots, a_n = l_n) \\ E' = [a_1 : l_1, \ldots, a_n : l_n] \\ S_1 = S[D_{T1}/l_1, \ldots, D_{Tn}/l_n] \\ v, E', S_1 \vdash \{ a_1 \leftarrow e_1; \ldots; a_n \leftarrow e_n; \} : v_n, S_2 \end{array}}{so, E, S \vdash \text{new } T : v, S_2}$$

Only the attributes are in scope (same as in typing）

Operational Semantics of Method Dispatch

- Consider the expression $e_0.f(e_1,…,e_n)$

- Informal semantics:
  1. Evaluate the arguments in order $e_1,…,e_n$
  2. Evaluate e_0 to the target object
  3. Let X be the dynamic type of the target object from e_0
  4. Fetch from X the definition of f (with n args.)
  5. Create n new locations and an environment that maps f's formal arguments to those locations
  6. Initialize the locations with the actual arguments
  7. Set self to the target object and evaluate f's body

Operational Semantics of Method Dispatch

$$\frac{\begin{array}{c} so, E, S \vdash e_1 : v_1 , S_1 \\ so, E, S_1 \vdash e_2 : v_2 , S_2 \\ \dots \\ so, E, S_{n-1} \vdash e_n : v_n , S_n \\ so, E, S_n \vdash e_0 : v_0, S_{n+1} \\ v_0 = X(a_1 = l_1,\dots, a_m = l_m) \\ impl(X, f) = (x_1,\dots, x_n, e_{body}) \\ l_{xi} = newloc(S_{n+1}) \text{ for } i = 1,\dots,n \\ E' = [x_1 : l_{x1}, \dots, x_n : l_{xn}, a_1 : l_1,\dots,a_m : l_m] \\ S_{n+2} = S_{n+1}[v_1/l_{x1},\dots,v_n/l_{xn}] \\ v_0 , E', S_{n+2} \vdash e_{body} : v, S_{n+3} \end{array}}{so, E, S \vdash e_0.f(e_1,\dots,e_n) : v, S_{n+3}}$$

Evaluate the arguments in order

← Evaluate e_0 to the target object

Create n new locations and an environment that maps f's formal arguments to those locations

← Initialize the locations with the actual arguments

← evaluate f's body

For a class A and a method f of A (possibly inherited) we write: (implementation mapping)

$impl(A, f) = (x_1, \dots, x_n, e_{body})$ where

- x_i are the names of the formal arguments
- e_{body} is the body of the method

Operational Semantics of Static Method Dispatch

$$\frac{\begin{array}{c} so, E, S \vdash e_1 : v_1 , S_1 \\ so, E, S_1 \vdash e_2 : v_2 , S_2 \\ \ldots \\ so, E, S_{n-1} \vdash e_n : v_n , S_n \\ so, E, S_n \vdash e_0 : v_0, S_{n+1} \\ v_0 = X(a_1 = l_1,\ldots, a_m = l_m) \\ impl(X, f) = (x_1,\ldots, x_n, e_{body}) \\ l_{xi} = newloc(S_{n+1}) \text{ for } i = 1,\ldots,n \\ E' = [x_1 : l_{x1}, \ldots, x_n : l_{xn}, a_1 : l_1,\ldots,a_m : l_m] \\ S_{n+2} = S_{n+1}[v_1/l_{x1},\ldots,v_n/l_{xn}] \\ v_0 , E', S_{n+2} \vdash e_{body} : v, S_{n+3} \end{array}}{so, E, S \vdash e_0.f(e_1,\ldots,e_n) : v, S_{n+3}}$$

$$\frac{\begin{array}{c} so, E, S \vdash e_1 : v_1 , S_1 \\ so, E, S_1 \vdash e_2 : v_2 , S_2 \\ \ldots \\ so, E, S_{n-1} \vdash e_n : v_n , S_n \\ so, E, S_n \vdash e_0 : v_0, S_{n+1} \\ v_0 = X(a_1 = l_1,\ldots, a_m = l_m) \\ impl(T, f) = (x_1,\ldots, x_n, e_{body}) \\ l_{xi} = newloc(S_{n+1}) \text{ for } i = 1,\ldots,n \\ E' = [x_1 : l_{x1}, \ldots, x_n : l_{xn}, a_1 : l_1,\ldots,a_m : l_m] \\ S_{n+2} = S_{n+1}[v_1/l_{x1},\ldots,v_n/l_{xn}] \\ v_0 , E', S_{n+2} \vdash e_{body} : v, S_{n+3} \end{array}}{so, E, S \vdash e_0@T.f(e_1,\ldots,e_n) : v, S_{n+3}}$$

Runtime Errors

$$\frac{\begin{array}{c}
so, E, S \vdash e_1 : v_1 , S_1 \\
so, E, S_1 \vdash e_2 : v_2 , S_2 \\
\ldots \\
so, E, S_{n-1} \vdash e_n : v_n , S_n \\
so, E, S_n \vdash e_0 : v_0, S_{n+1} \\
v_0 = X(a_1 = l_1, \ldots, a_m = l_m) \\
impl(X, f) = \text{ not defined?} \\
l_{xi} = newloc(S_{n+1}) \text{ for } i = 1,\ldots,n \\
E' = [x_1 : l_{x1}, \ldots, x_n : l_{xn}, a_1 : l_1, \ldots, a_m : l_m] \\
S_{n+2} = S_{n+1}[v_1/l_{x1}, \ldots, v_n/l_{xn}] \\
v_0 , E', S_{n+2} \vdash e_{body} : v, S_{n+3}
\end{array}}{so, E, S \vdash e_0.f(e_1,\ldots,e_n) : v, S_{n+3}}$$

Cannot happen in a well-typed program (Type safety theorem)

Runtime Errors (Cont.)

- There are some runtime errors that the type checker does not try to prevent
  - A dispatch on void
  - Division by zero
  - Substring out of range
  - Heap overflow

- In such case the execution must abort gracefully
  - With an error message, not with segfault

Conclusion

- Operational rules are very precise
  - Nothing that matters is left unspecified
- Operational rules contain a lot of details
  - But not too many details, no stack or heap
  - Read them carefully
- Most languages do not have a well specified operational semantics
- When portability is important an operational semantics becomes essential
  - But not always using the notation we used for Cool

Reading

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