

Parallel Continuous Chain-of-Thought with Jacobi Iteration

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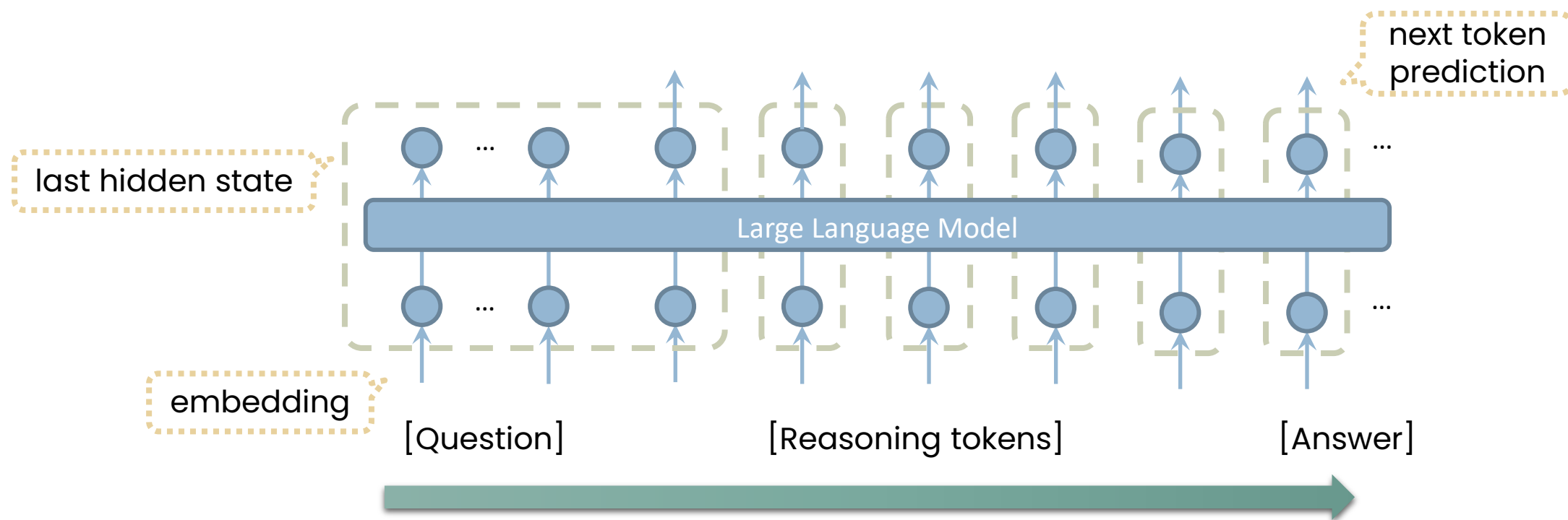
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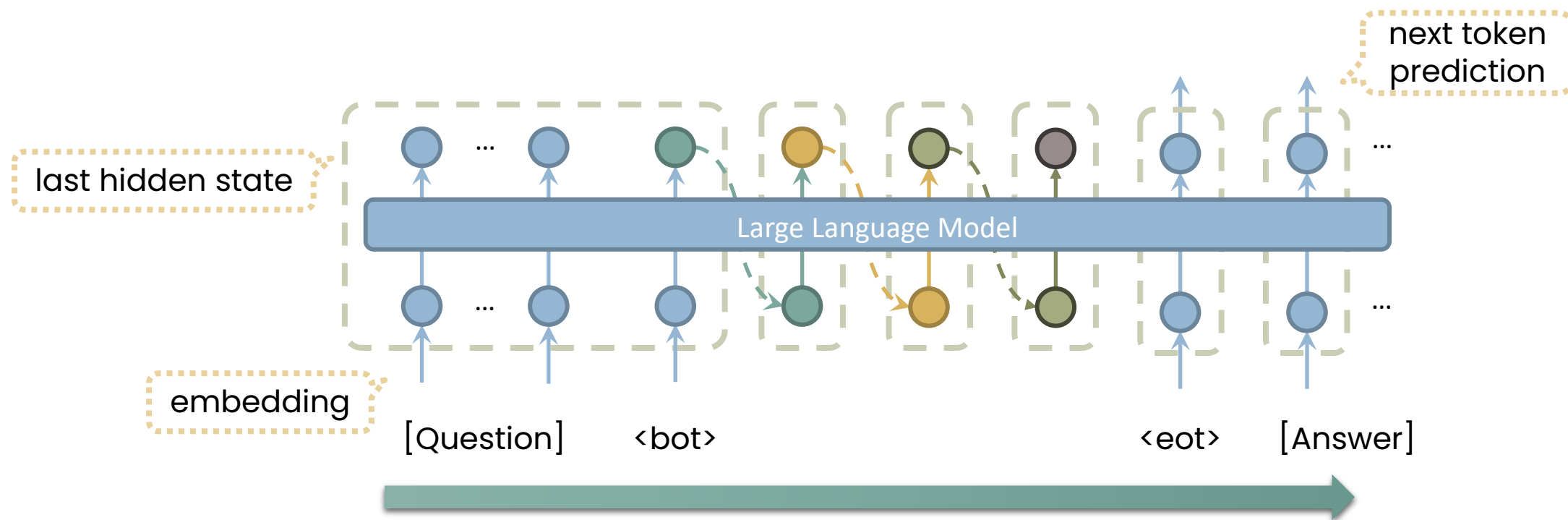
Chain-of-Thought

Parallel Continuous Chain-of-Thought with Jacobi Iteration



Continuous Chain-of-Thought

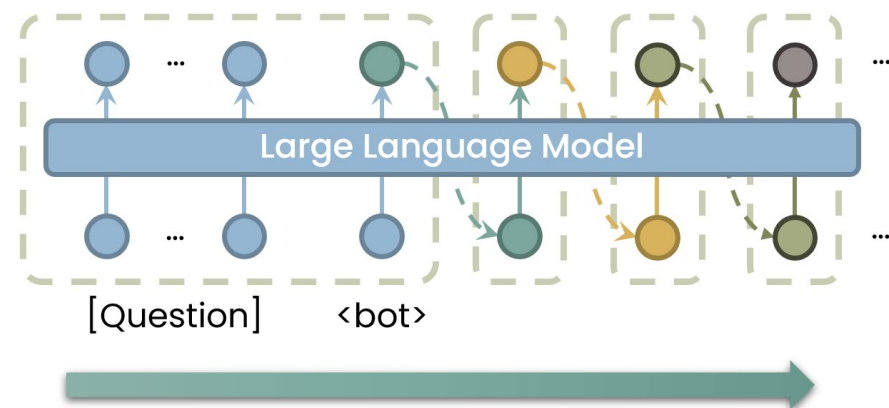
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Continuous Chain-of-Thought

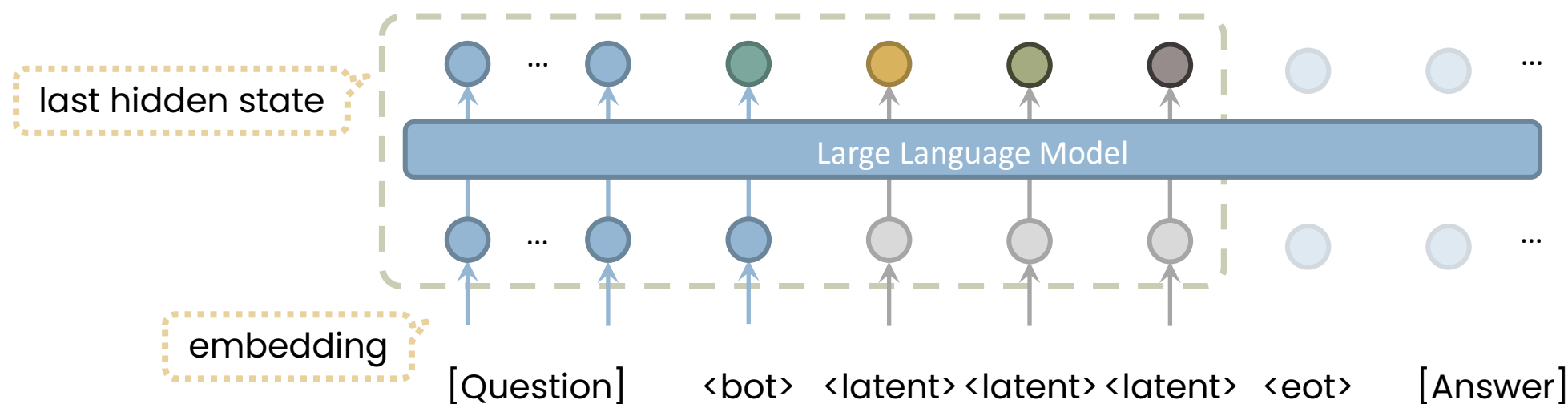
Parallel Continuous Chain-of-Thought with Jacobi Iteration

- by performing implicit reasoning with continuous vectors...
 - ...it saves reasoning tokens
 - ...but the sequential dependency spoils parallel training



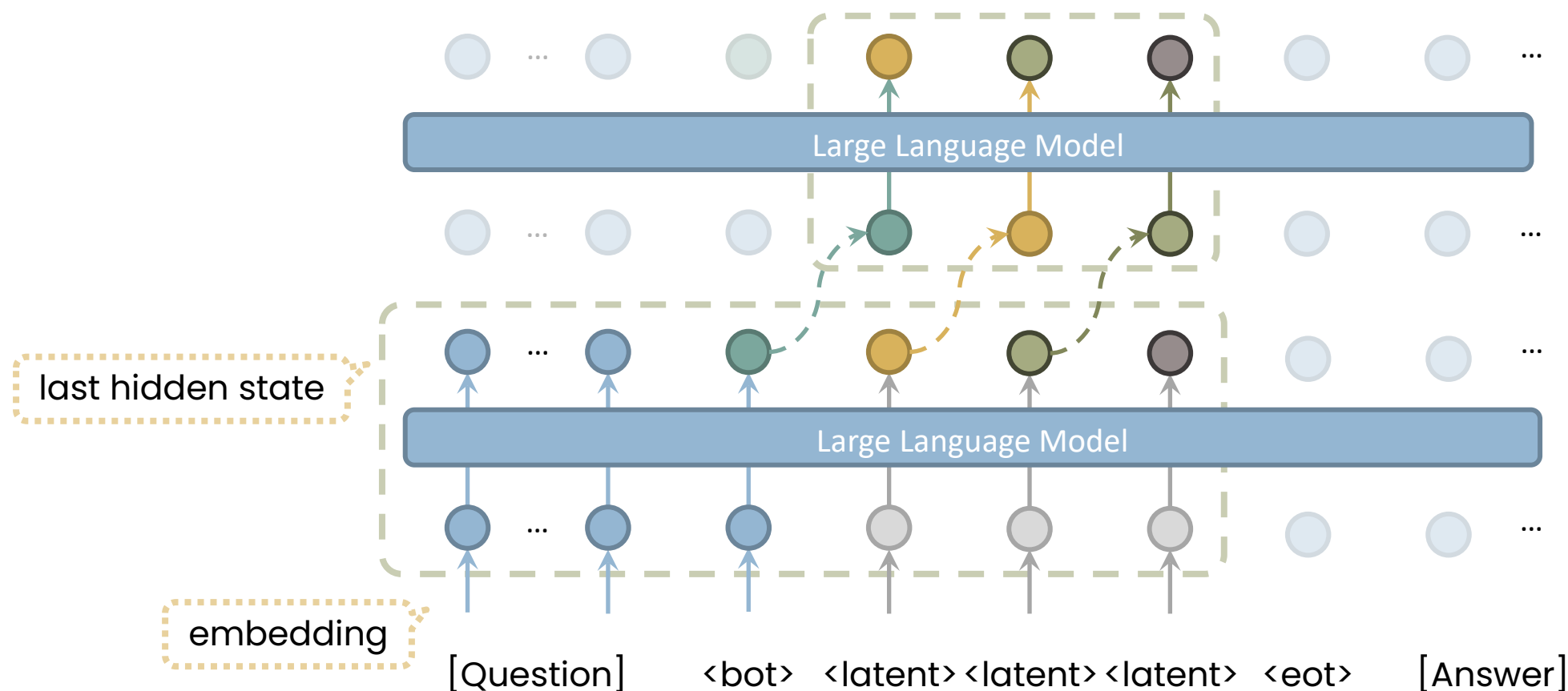
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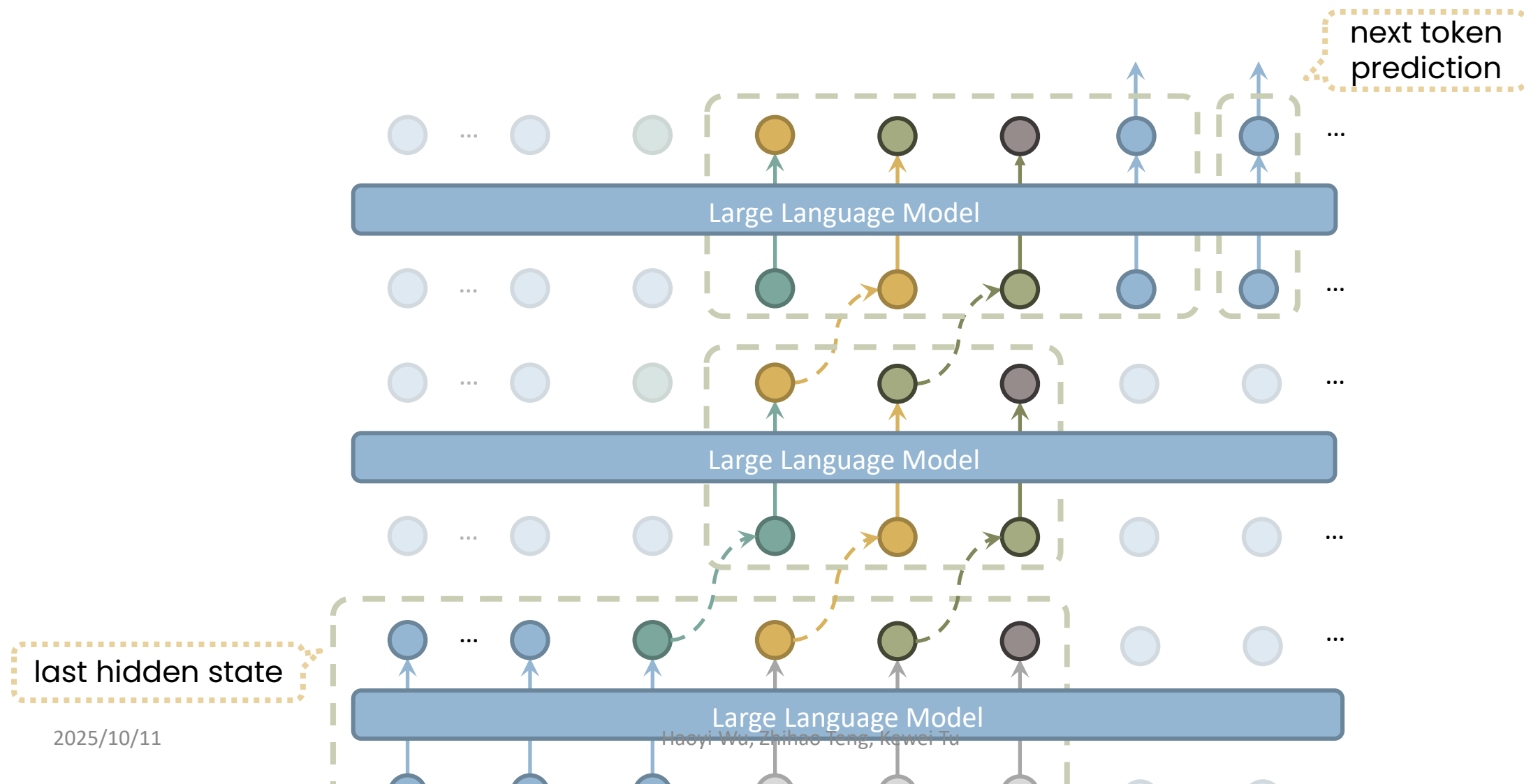
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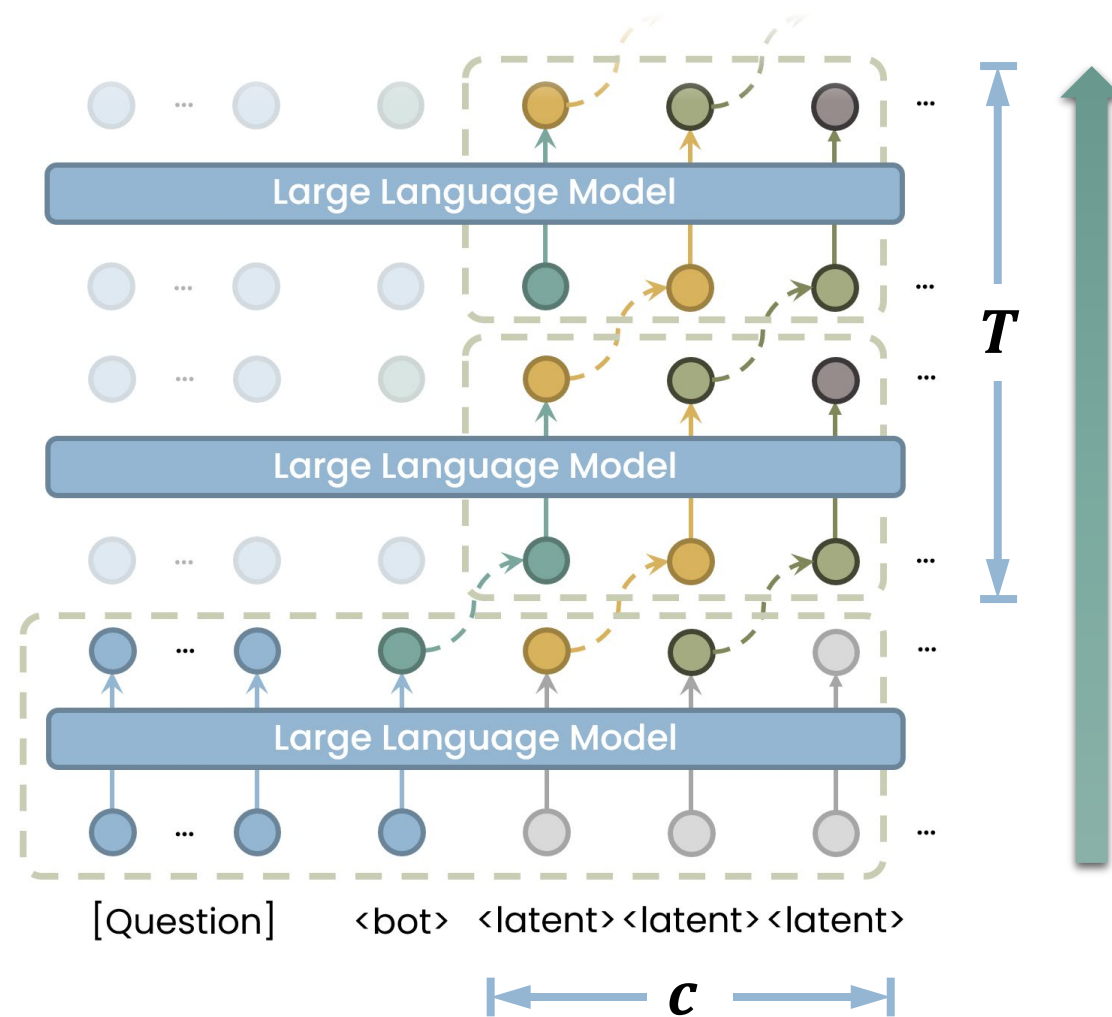
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Parallel Continuous Chain-of-Thought

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- PCCoT performs non linear Jacobi iteration on latent thought tokens
- Two hyperparameters:
 - c : # of latent thought tokens;
 - T : # of extra iterations.



Relations to Existing Approaches

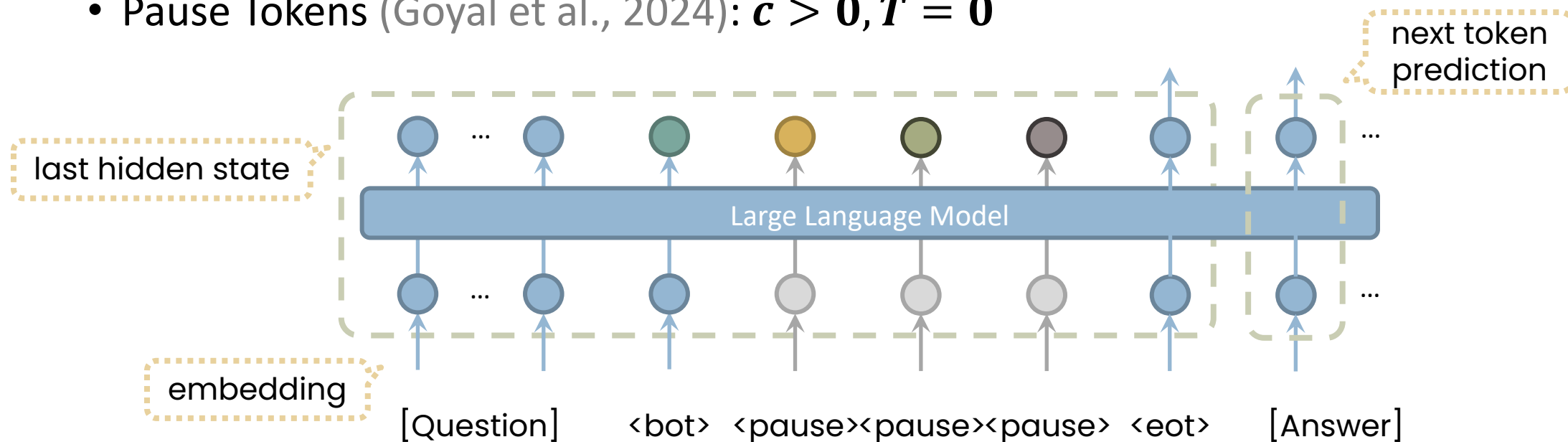
Parallel Continuous Chain-of-Thought with Jacobi Iteration

- with different settings of $\mathbf{c}$ and $\mathbf{T}$, PCCoT can be reduced to these existing approaches:
 - Implicit CoT (iCoT) (Deng et al., 2024): $\mathbf{c} = \mathbf{0}$

Relations to Existing Approaches

Parallel Continuous Chain-of-Thought with Jacobi Iteration

- with different settings of $\mathbf{c}$ and $\mathbf{T}$, PCCoT can be reduced to these existing approaches:
 - Implicit CoT (iCoT) (Deng et al., 2024): $\mathbf{c} = \mathbf{0}$
 - Pause Tokens (Goyal et al., 2024): $\mathbf{c} > \mathbf{0}, \mathbf{T} = \mathbf{0}$

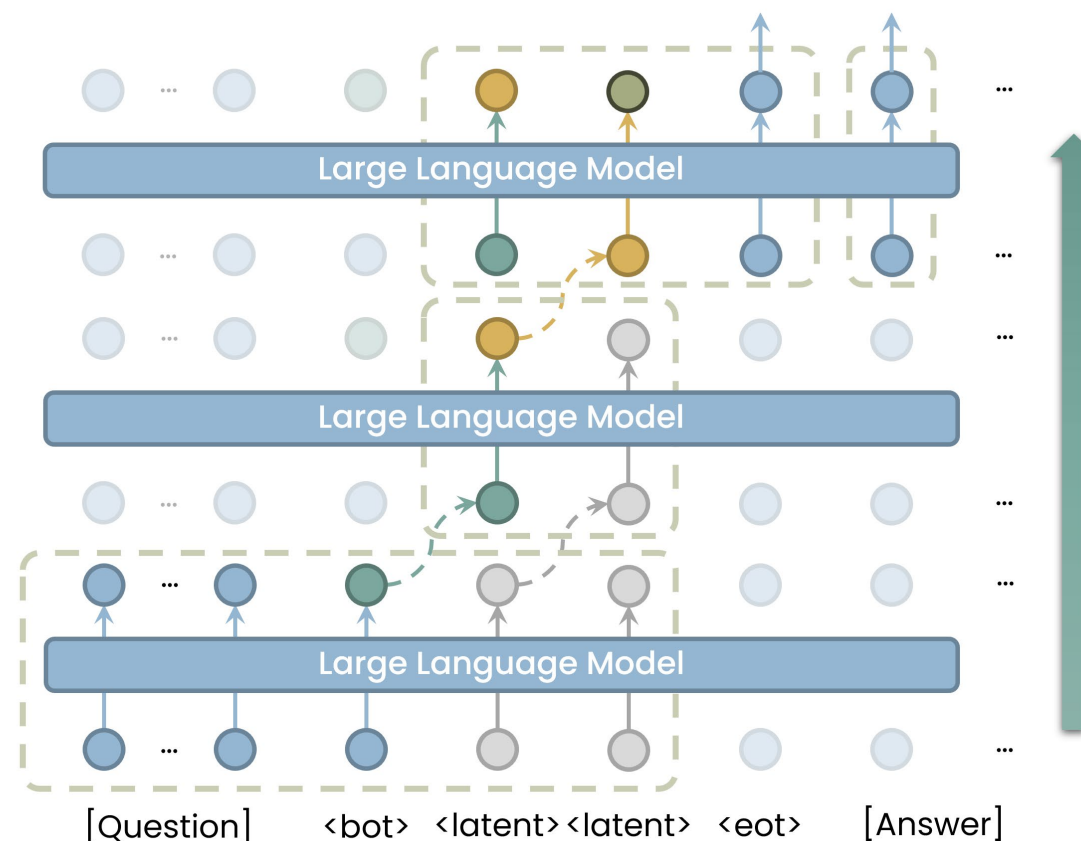
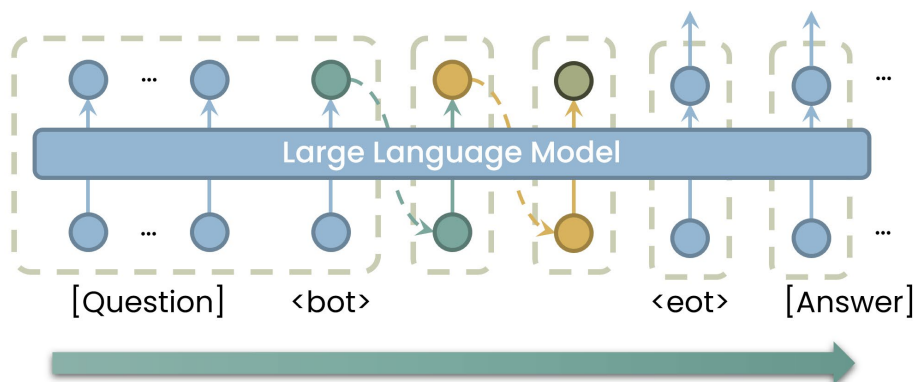


Relations to Existing Approaches

Parallel Continuous Chain-of-Thought with Jacobi Iteration

- with different settings of c and T , PCCoT can be reduced to these existing approaches:

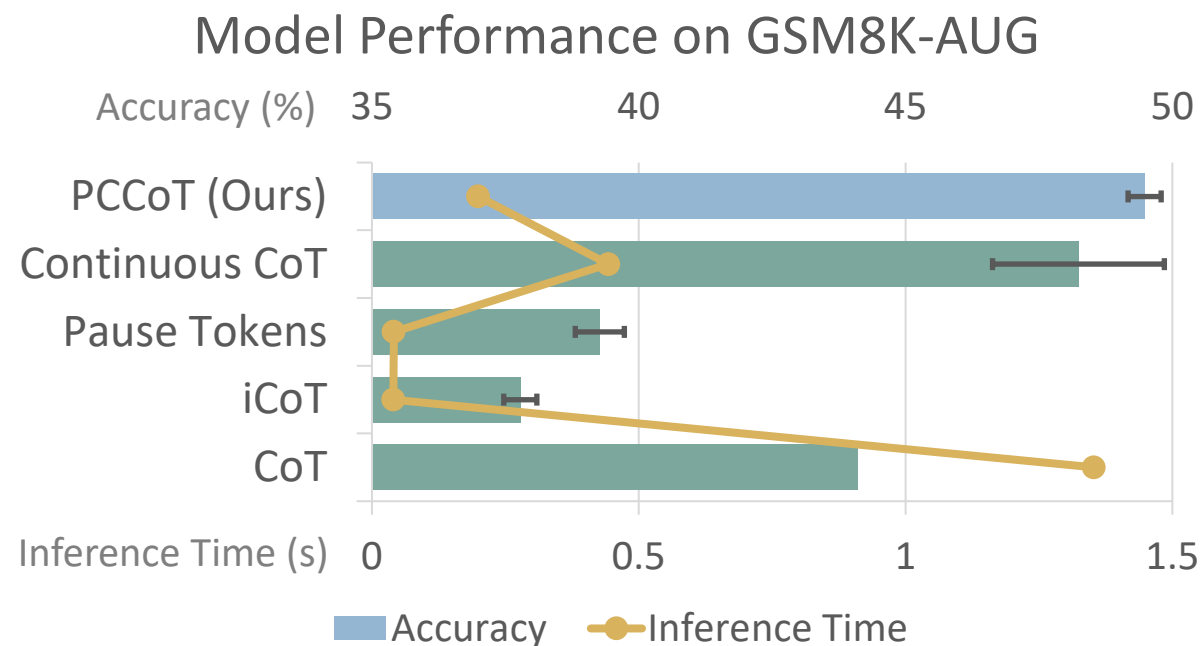
- Implicit CoT (iCoT): $c = 0$
- Pause Tokens: $c > 0, T = 0$
- Continuous CoT (Hao et al., 2024):
 $T \geq c > 0$



Experiments

Parallel Continuous Chain-of-Thought with Jacobi Iteration

- Proper choice of c and T speeds up the reasoning process by a large scale without sacrificing the performance.

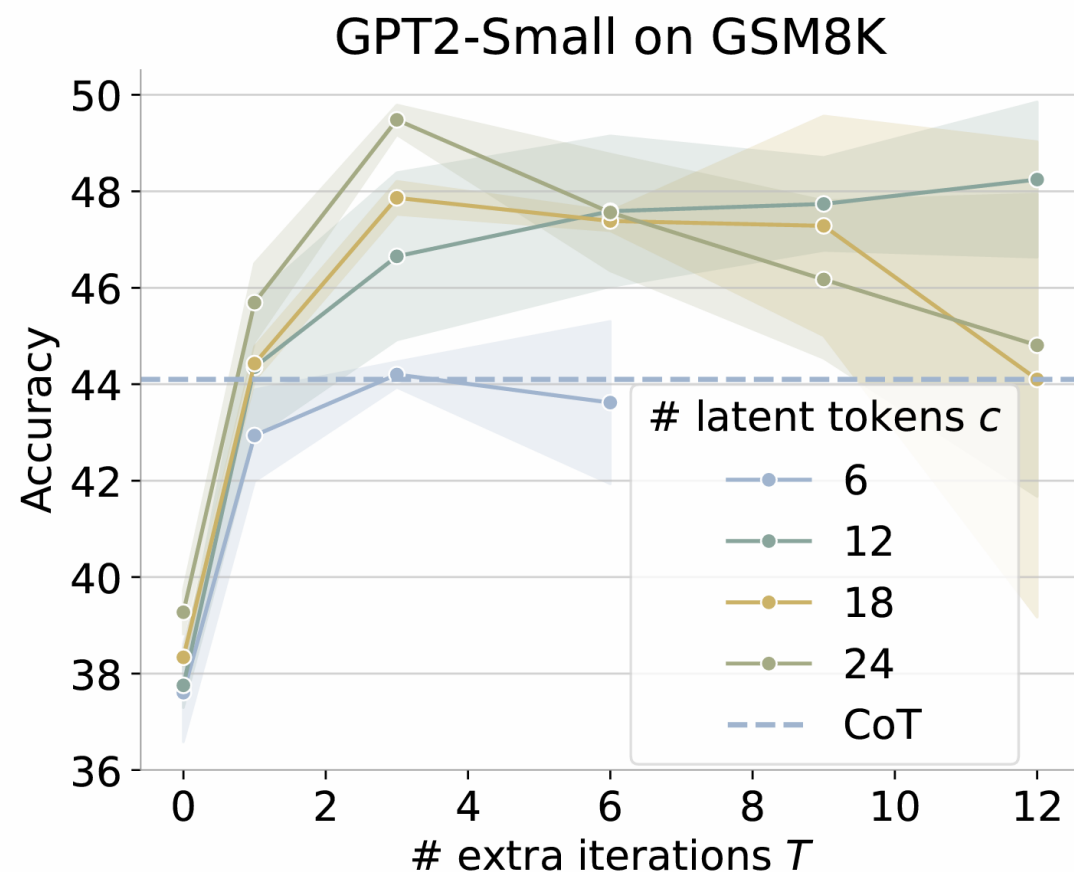


* Experiment on GPT-2 Small, finetuned with LoRA on GSM8K-AUG. The results are averaged over 3 random runs with standard deviations as error bars. The inference time is measured with a batch size of 100. The training runs on 2 H800 GPUs and the inference runs on 1 A6000 GPU. Model training follows that of CODI (Shen et al. 2025).

Experiments

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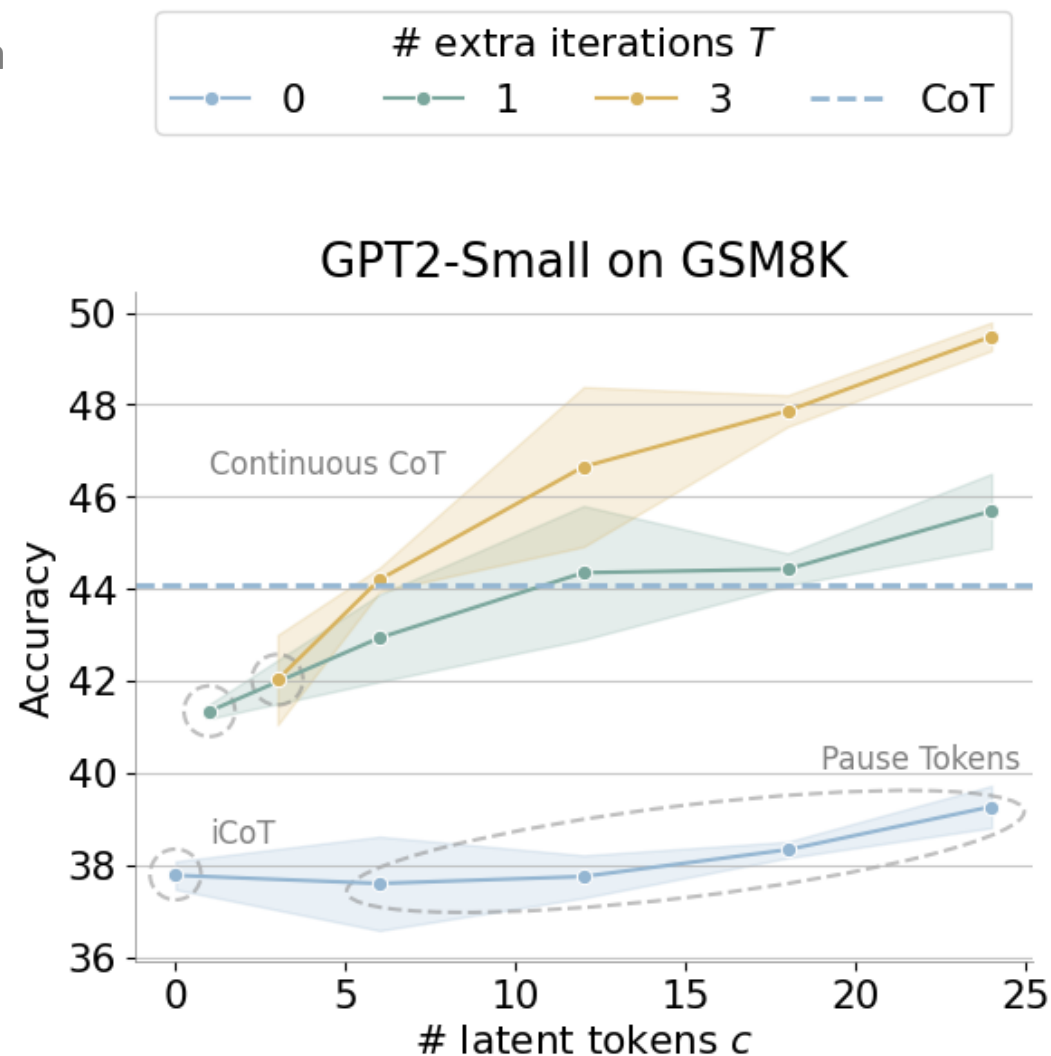
- Proper choice of c and T speeds up the reasoning process by a large scale without sacrificing the performance.
 - Large $T \not\Rightarrow$ performance improvement
 - Large $T \Rightarrow$ large standard deviation



Experiments

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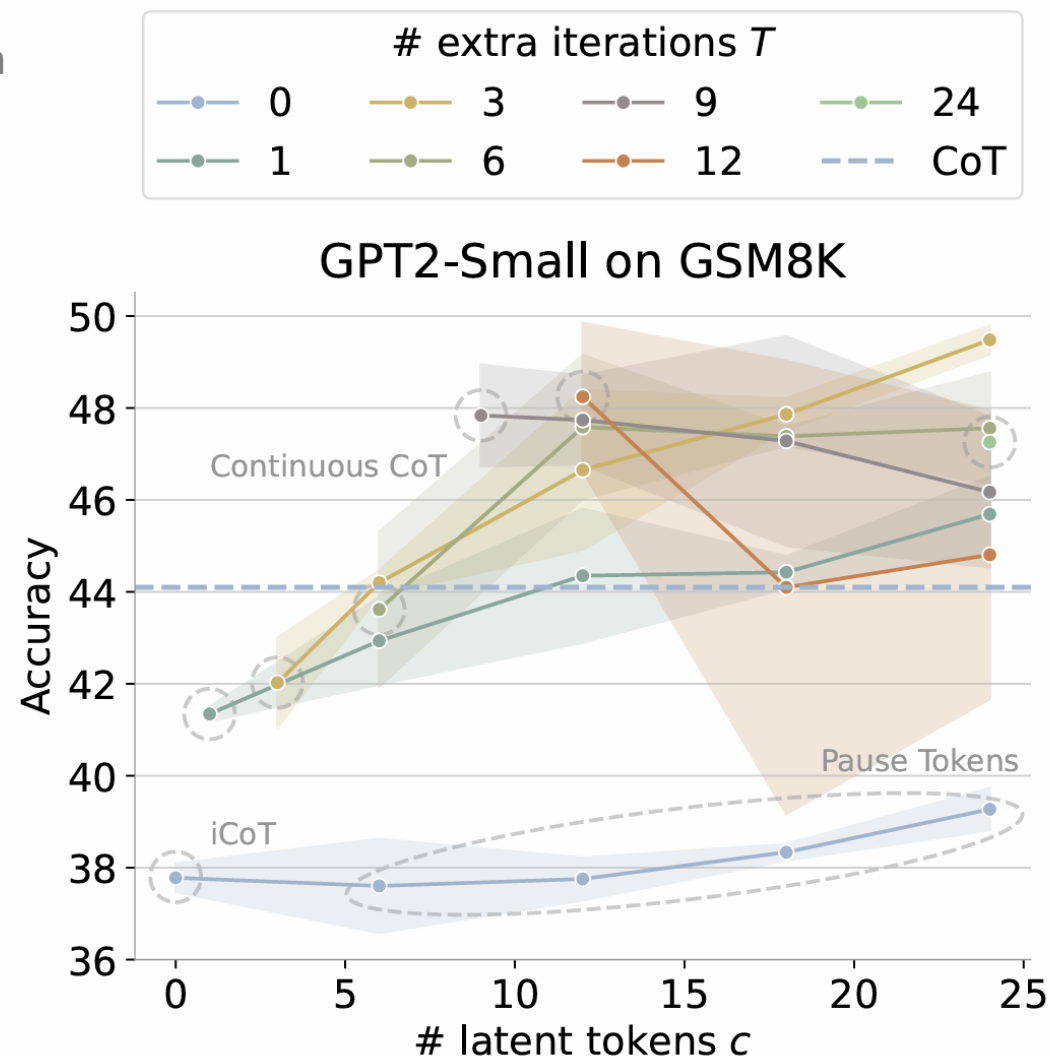
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 - Large $T \not\Rightarrow$ performance improvement
 - Large $T \Rightarrow$ large standard deviation
 - Large $c \Rightarrow$ stable superior performance



Experiments

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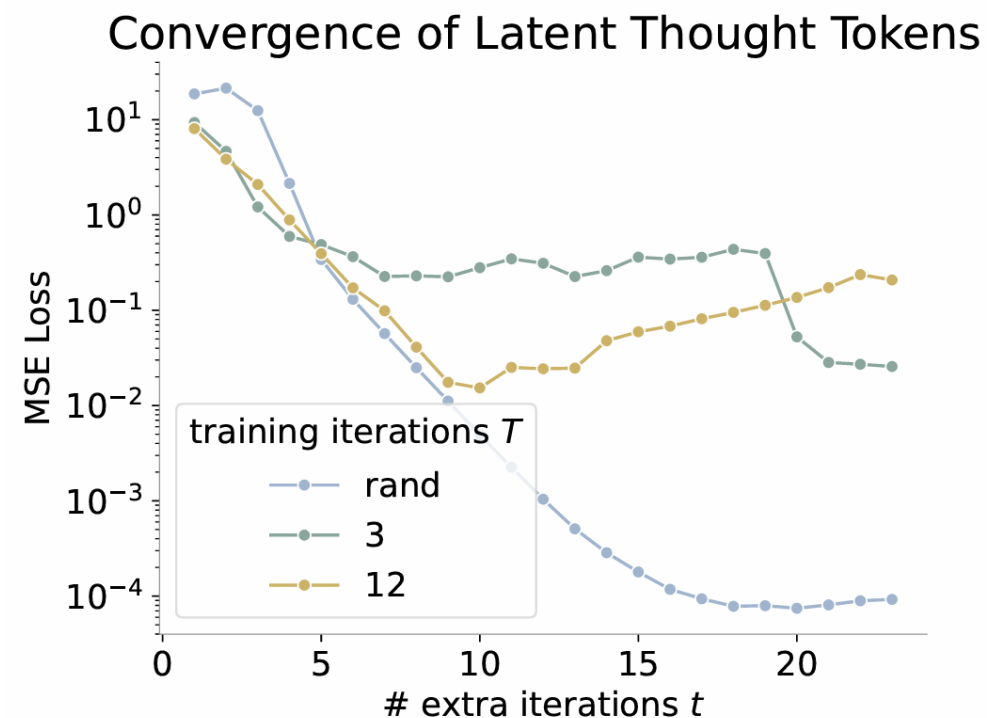
- Proper choice of c and T speeds up the reasoning process by a large scale without sacrificing the performance.
 - Large $T \not\Rightarrow$ performance improvement
 - Large $T \Rightarrow$ large standard deviation
 - Large $c \Rightarrow$ stable superior performance if T is small



Analysis

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- Surprisingly, the latent thought tokens do not converge
 - Despite the convergence guarantee when $T \geq c$



* MSE of the latent thought tokens before and after the t th extra iteration. “rand” means the model is randomly initialized. Other models are trained with $c = 24$ and different T . The model is tested on random samples from the test set of GSM8K.

Thanks

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github.com/whyNLP/PCCoT



faculty.sist.shanghaitech.edu.cn/faculty/tukw/



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