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Dynamic Stochastic Computing for Digital Signal Processing Applications

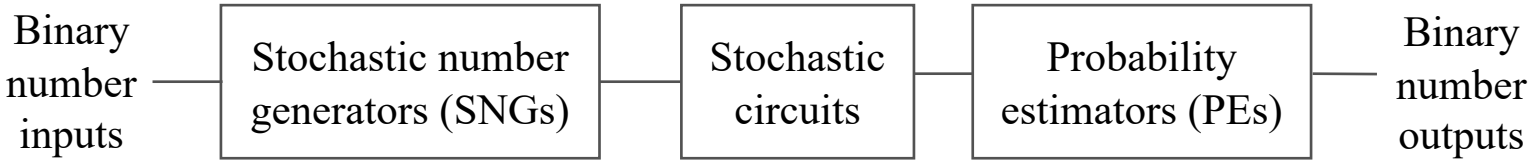
Siting Liu and Jie Han
Department of Electrical and Computer Engineering
University of Alberta
Edmonton, AB, Canada

Outline

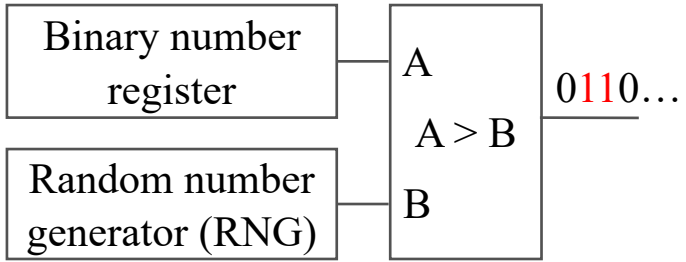
- Introduction of stochastic computing
- Dynamic stochastic computing systems
- DSP using dynamic stochastic computing
- Hardware assessment
- Comparison with $\Delta - \Sigma$ modulation
- Conclusion
- References

Stochastic Computing

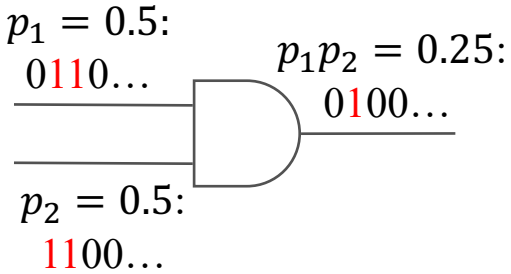
- In stochastic computing (SC), information is encoded and processed by streams of random bits [1].



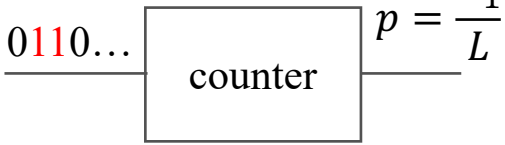
A stochastic computing system.



An SNG.



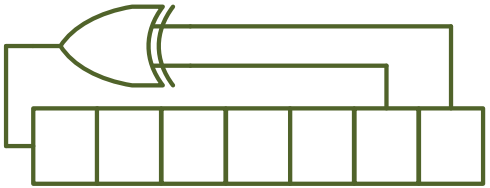
A unipolar stochastic multiplier.



A probability estimator.

Limitations of Stochastic Computing

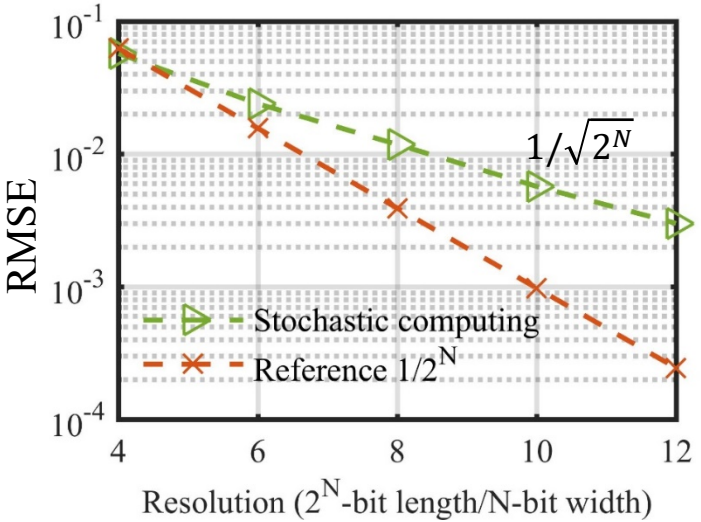
- Low accuracy
  - Random fluctuations



Linear feedback shift register (LFSR) as a random number generator (RNG).

- Potential solution: dynamic stochastic computing (DSC)
  - An extreme case: 1-bit SC with high accuracy.

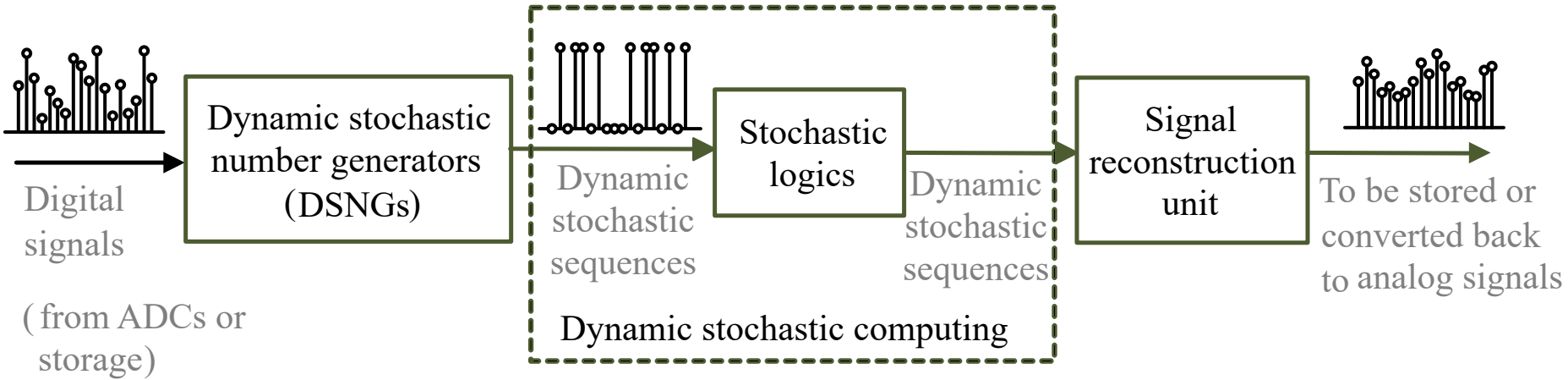
- High latency



Accuracy of a unipolar stochastic multiplier using pseudorandom sequences.

Dynamic Stochastic Computing Systems

- In a dynamic stochastic computing (DSC) system, the stochastic sequence can encode a consistently varying signal instead of a static number as in conventional stochastic computing systems.



A dynamic stochastic computing (DSC) system.

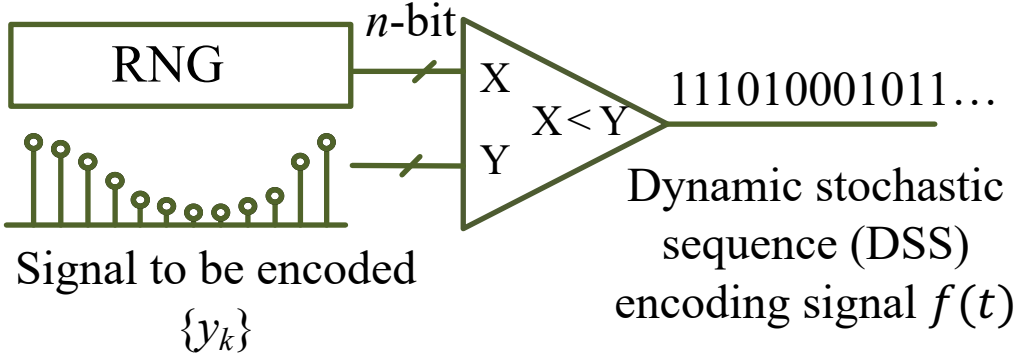
Generation of Dynamic Stochastic Sequences

❑ Definition: dynamic stochastic sequence (DSS):

A DSS $\{A_k\}$ encoding $f(t)$ satisfies that the kth bit in the random binary sequence has the expectation,

$$\mathbb{E}[A_k] = f(kT) = y_k.$$

*T is the sampling period and is used for the rest of the slides.



- A dynamic SNG (DSNG) can be realized by comparing each sampling point with a uniform random number generated by the RNG.
- A DSS can also be generated by a $\Delta - \Sigma$ modulator [6].

An example of a DSS

❑ Dynamic stochastic sequence (DSS):

A DSS $\{A_k\}$ encoding $f(t)$ satisfies that the kth bit in the random binary sequence has the expectation,

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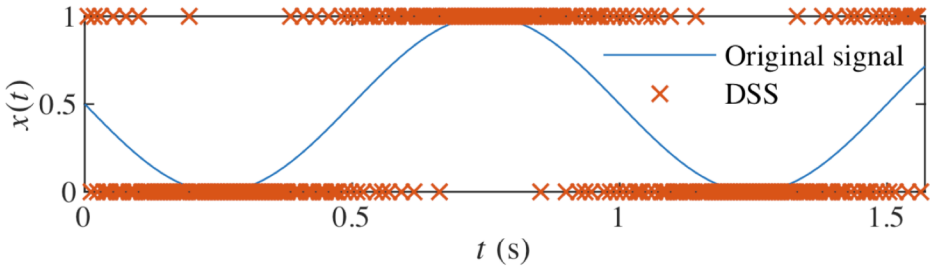
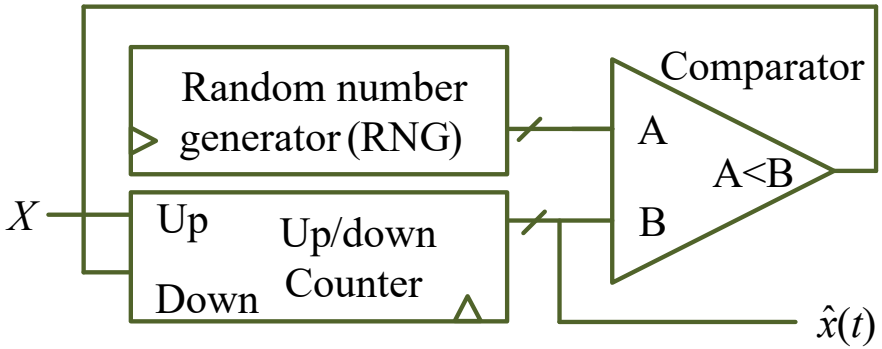


A DSS encoding a 1-Hz sine wave.
The DSS is decimated for a clear view.

A higher frequency of '1' is observed in the DSS when the signal is close to 1 and vice versa.

Reconstruction of Dynamic Stochastic Sequences

- ADaptive DIgital Element (ADDIE) [1, 2]



An ADDIE.

It implements an exponential smoothing function. There are also other methods (low-pass filtering) but generally induce more hardware cost.

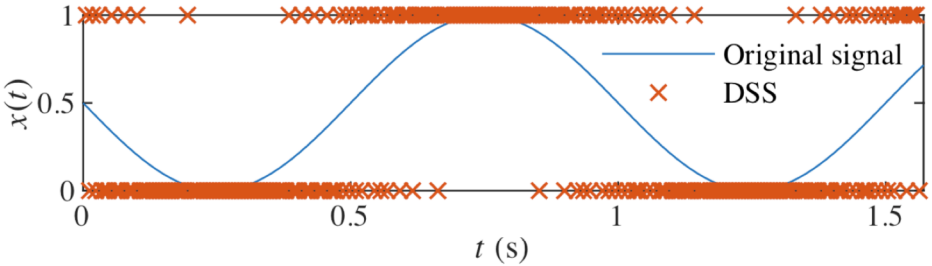
$$\hat{x}(t) \approx \frac{1}{2^N} \sum_{k=0}^{i} \left[\left(\frac{2^N - 1}{2^N} \right)^k \mathbb{E}[X_{i-k}] \right]$$

*$t = (i+1)T$; N is the bit-width of the counter; T is the sampling period.
*Proof is available in the paper.

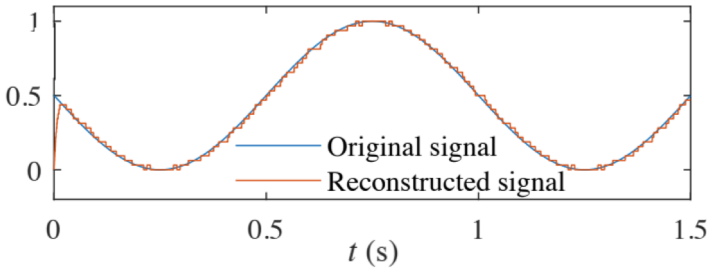
Recent information takes larger weight while history information takes smaller weight.

Signal Reconstruction using ADDIE

- ADaptive DIgital Element (ADDIE) [1, 2]



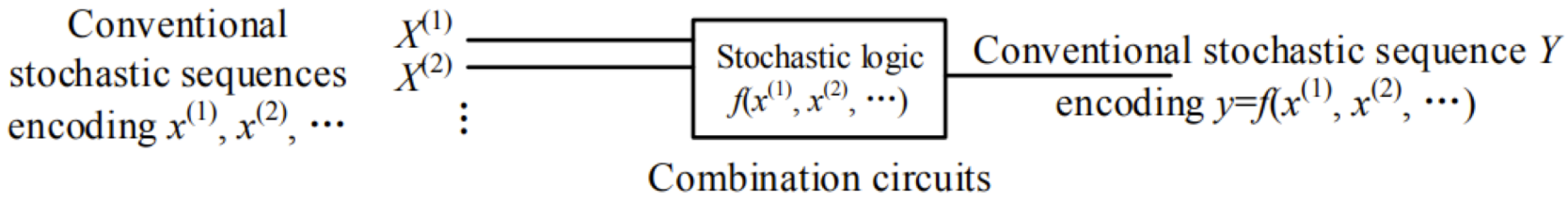
(a)



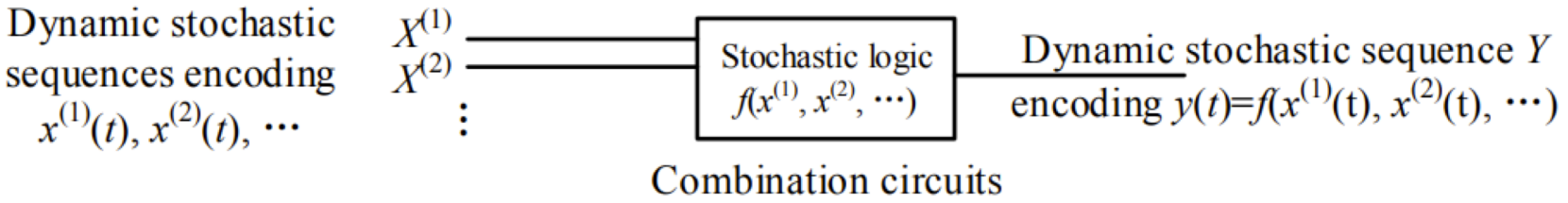
(b)

A DSS reconstructed by an ADDIE (a) the DSS; (b) reconstructed signal.

Dynamic Stochastic Circuit for Digital Signal Processing

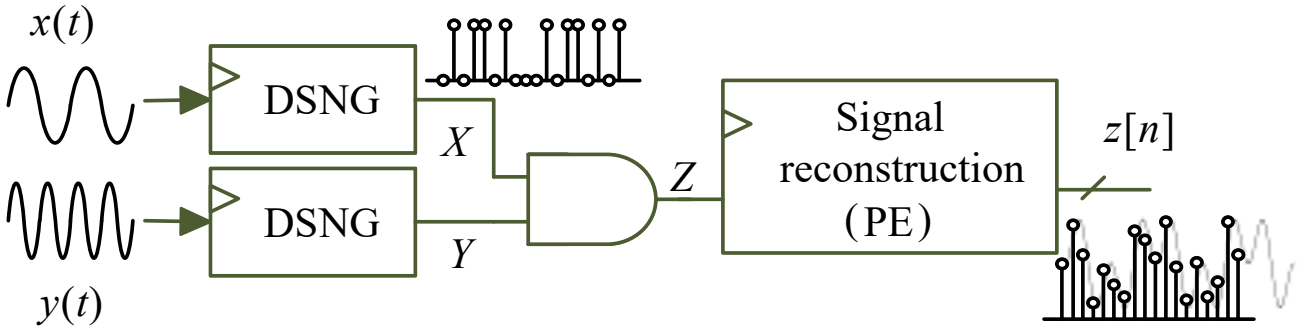


- Conventional stochastic computing for digital signal processing.
  A long sequence is typically used to encode one value..



- Dynamic stochastic computing for digital signal processing.
  A dynamic stochastic sequences is used to encode a signal.

Stochastic Circuit - Frequency Mixing



Frequency mixing using a stochastic multiplier.

- Let X_k, Y_k and Z_k be the kth bits in the dynamic sequences. X_k and Y_k are independent, then

$$\mathbb{E}[Z_k] = \mathbb{E}[X_k]\mathbb{E}[Y_k] = x(kT)y(kT).$$

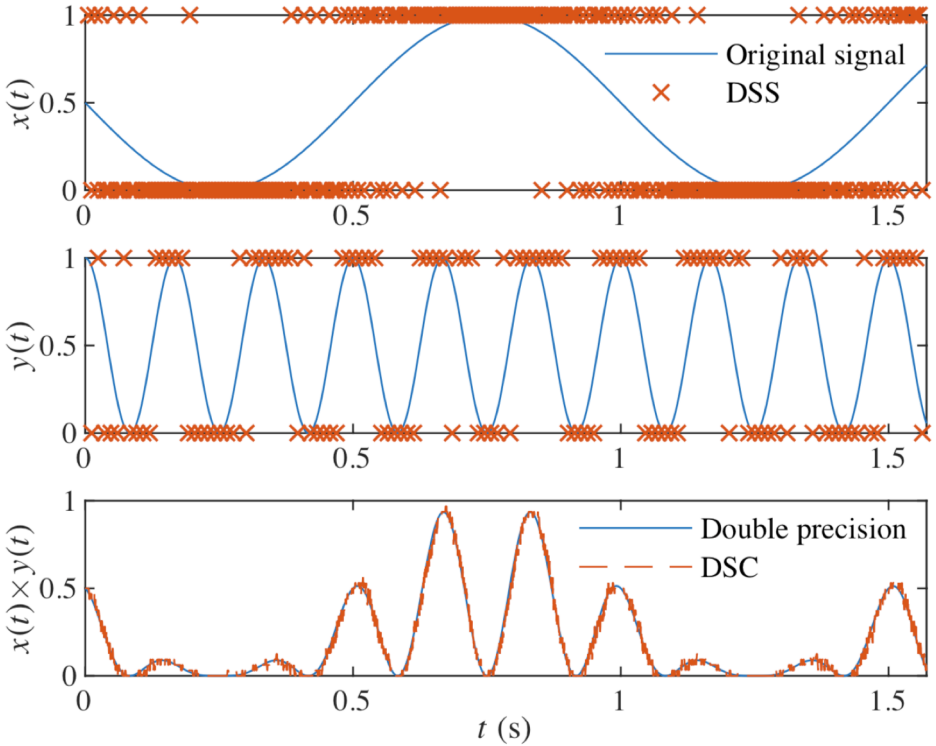
- So the output sequence encodes

$$z(t) = x(t)y(t).$$

Frequency Mixing Experiment

❑ Experiment setup:

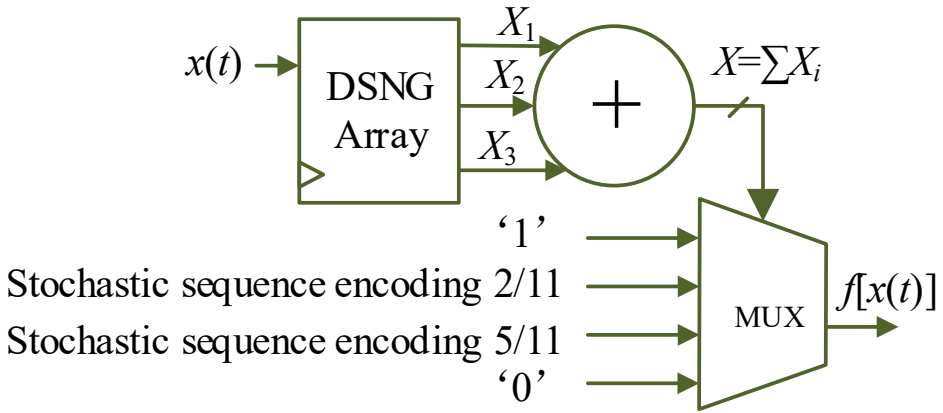
- $x(t)$: 1 Hz sinusoidal signal;
- $y(t)$: 6 Hz sinusoidal signal.
- $x(t)$ and $y(t)$ are sampled by 2^{16} Hz sampling frequency and then converted to DSS's.
- The output sequence is reconstructed by a 5-bit ADDIE.
- Reconstructed signal is compared with double-precision result.



A frequency mixer multiplying two sine waves.

Stochastic Circuit – Function Approximation

❑ Function approximation using multiplexing circuit

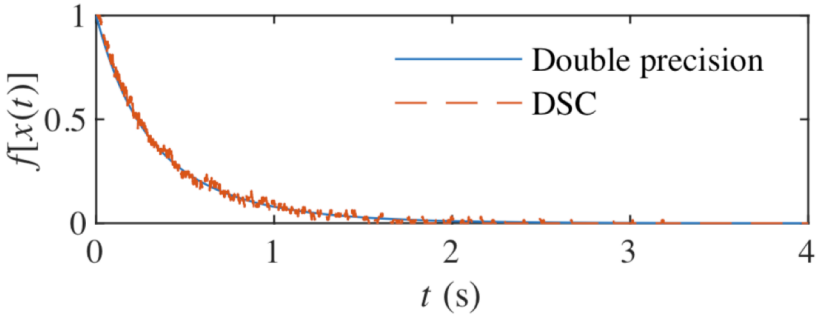


- The multiplexing circuit originally computes $f(x) = 1/11(2x^3 + 3x^2 + 6x^1)$ [2].
- If the input sequence encodes $x(t) = e^{-2t}$, the multiplexing circuit calculates function composition $f[x(t)] = 1/11(2e^{-6t} + 3e^{-4t} + 6e^{-2t})$.

Function Approximation Experiment

❑ Experiment setup:

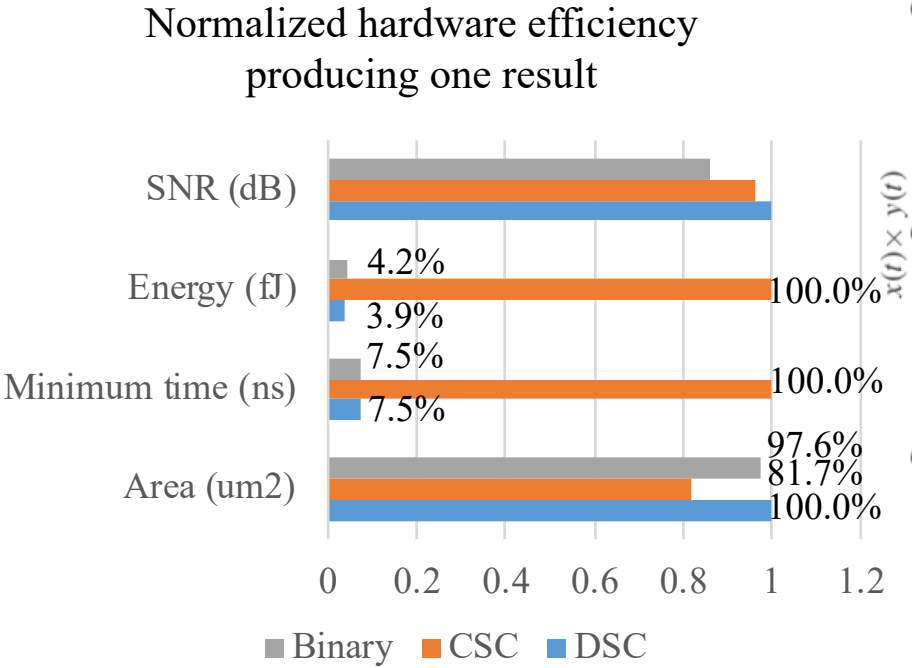
- $x(t) = e^{-2t}$;
- $x(t)$ is sampled by 2^{16} Hz sampling frequency and then converted to DSS's.
- Reconstructed signal is compared with double-precision result. For the double-precision result, it is assumed that e^{-2t} is given. Only multipliers and adders are used to compute the function.

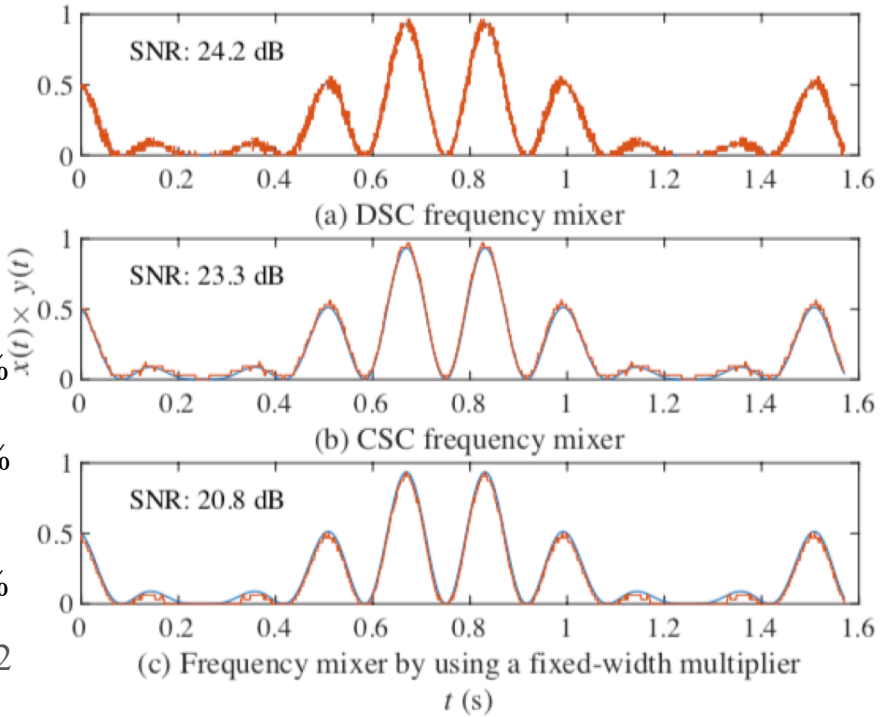


Approximation of function composition $f[x(t)] = 1/11(2e^{-6t} + 3e^{-4t} + 6e^{-2t})$ using DSC.

Hardware Evaluation for Frequency Mixing

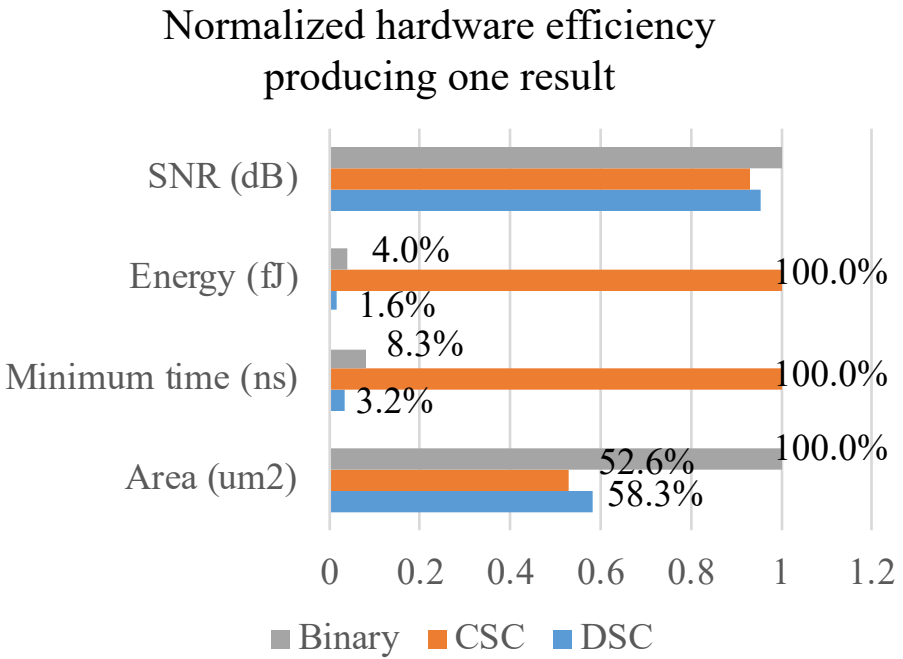
- Frequency mixing

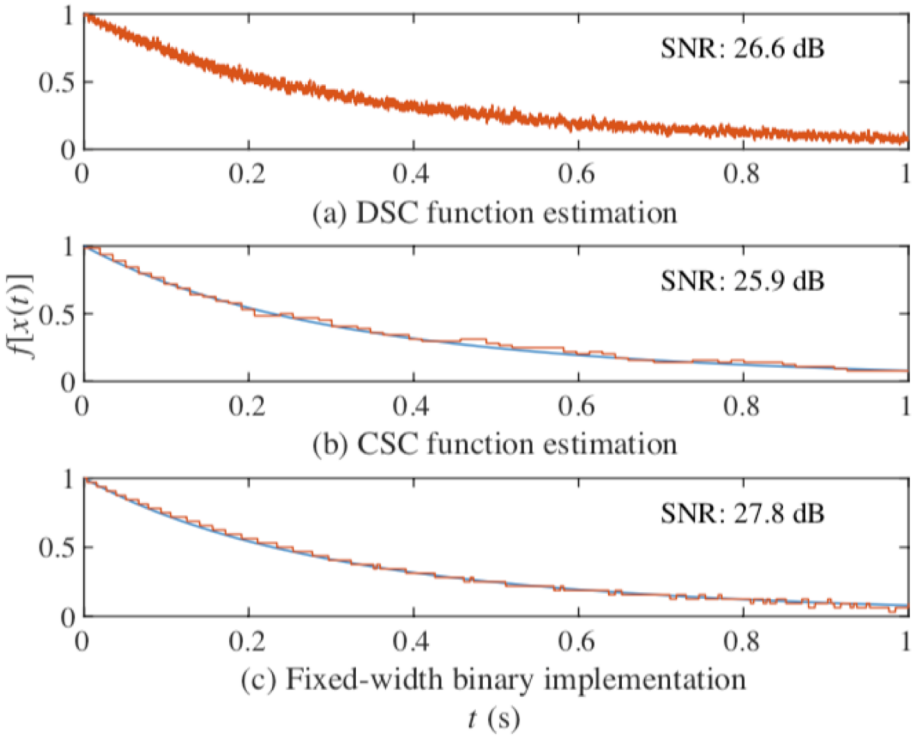




Hardware Evaluation for Function Approximation

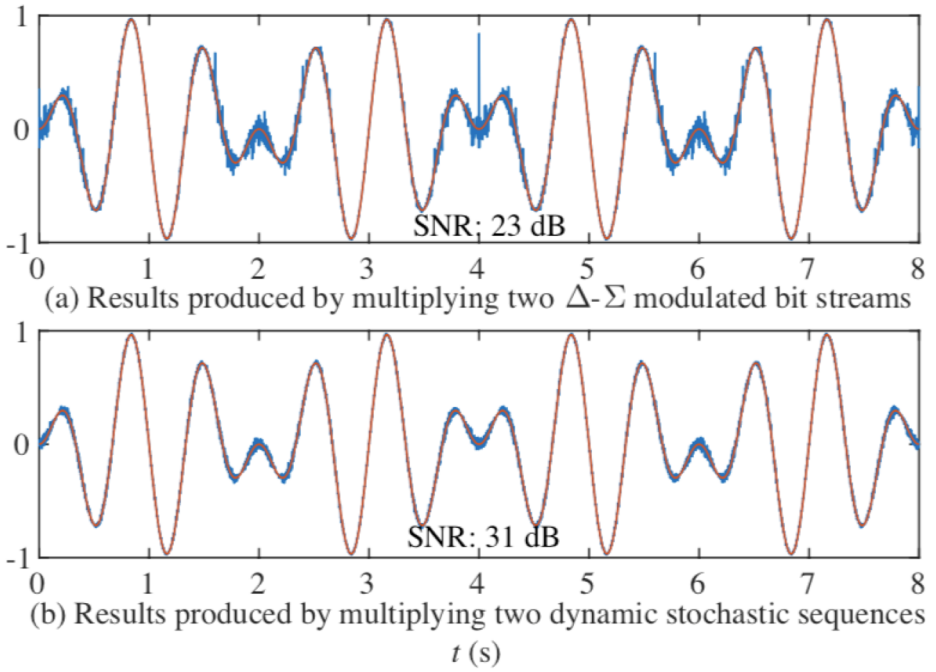
❑ Function estimation





Comparison with $\Delta - \Sigma$ Modulation

- DSS's and $\Delta - \Sigma$ modulated signals are similar since the generation of the streams can both be considered as Bernoulli processes [3,4]. However, $\Delta - \Sigma$ modulated signals may suffer from correlation issues [5].
- Frequency mixing using two (a) $\Delta - \Sigma$ modulated signals; (b) DSS's.



Conclusion

- Dynamic stochastic computing (DSC) is proposed;
- Dynamic stochastic sequences (DSS's) are used to encode a signal instead of a static number in DSC;
- Efficient digital signal processing, such as frequency mixing and function approximation, is enabled by using simplistic stochastic logic;
- A speedup and an energy efficiency improvement by more than 13 and 25 times for frequency mixing, 31 and 62 times for function approximation are achieved, compared to conventional stochastic computing

References

[1] Gaines, Brian R. "Stochastic computing systems." *Advances in information systems science*. Springer, Boston, MA, 1969. 37-172.

[2] Qian, Weikang, et al. "An architecture for fault-tolerant computation with stochastic logic." *IEEE transactions on computers* 60.1 (2010): 93-105.

[3] Saraf, Naman, et al. "IIR filters using stochastic arithmetic." *Design, Automation & Test in Europe Conference & Exhibition (DATE)*. IEEE, 2014.

[4] Lipasti, Mikko, and Carly Schulz. "End-to-end stochastic computing." *Proceedings of the 1st Workshop on Pioneering Processor Paradigms*. 2017.

[5] Gonzalez-Guerrero, Patricia, Xinfei Guo, and Mircea Stan. "SC-SD: Towards Low Power Stochastic Computing using Sigma Delta Streams." *IEEE International Conference on Rebooting Computing (ICRC)*. IEEE, 2018.

[6] V. da Fonte Dias. "Sigma-delta signal processing." *Proceedings of IEEE International Symposium on Circuits and Systems-ISCAS'94*. Vol. 5. IEEE, 1994.

Thank you for your attention.