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Can Stochastic Computing Truly Tolerate Bit Flips?

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Outline

- Background: stochastic computing & stochastic circuits
- Bit-flip model and bit-flip error recovery of stochastic circuits
- Experiments & results
- Application
- Conclusion

Background: stochastic computing

- In stochastic computing (SC), information is encoded and processed by streams of random bits. The probability of 1 (***p***) in the bit stream is used to represent the value (***x***).

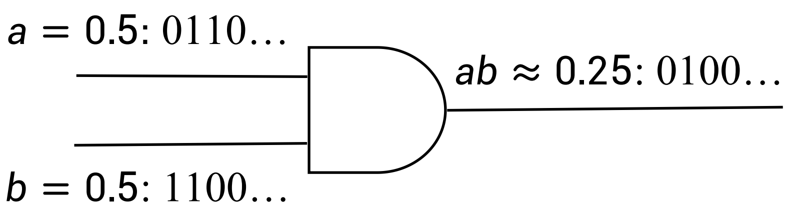
  - Binary representation: $(01000101)_2 \rightarrow 69$
  - Stochastic computing (SC): $01000101 \begin{cases} 3/8 & (unipolar) \\ -1/4 & (bipolar) \end{cases}$ $\quad x = 2p - 1$

$(00000101)_2 \rightarrow 5$ $\qquad$ $00000101 \begin{cases} 2/8 & (unipolar) \\ -2/4 & (bipolar) \end{cases}$

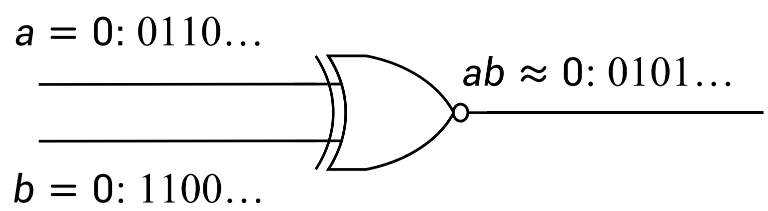
More resilient to bit flip errors

Background: stochastic circuits

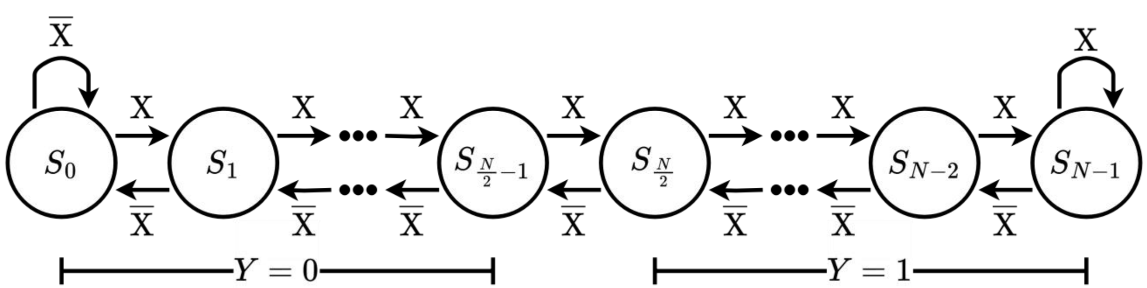
- Compared to conventional arithmetic circuits, the SC circuits are much simpler[1].



A unipolar stochastic multiplier.



A bipolar stochastic multiplier.



Stochastic FSM implementing tanh function

$$Y_i = \begin{cases} 0 \; when \; 0 \leq S_i \leq \frac{N}{2} - 1 \\ 1 \; when \; \frac{N}{2} - 1 < S_i \leq N \end{cases}$$

$$y = \frac{\left(\frac{1+x}{1-x}\right)^{\frac{N}{2}} - 1}{\left(\frac{1+x}{1-x}\right)^{\frac{N}{2}} + 1} = Stanh\left(\frac{N}{2}x\right)$$

$$\approx \frac{e^{\frac{N}{2}x} - e^{-\frac{N}{2}x}}{e^{\frac{N}{2}x} + e^{-\frac{N}{2}x}} = \tanh\left(\frac{N}{2}x\right)$$

More resilient to bit flip errors, based on empirical studies[2]

Bit flip models of stochastic sequences

- Assume bit flips are Bernoulli events with probability P_E and they are indepedent
- Stochastic sequence with probability P_X, representing x
- Probability considering bit flips P_X'

$$x' = P_X' = P_X(1 - P_E) + (1 - P_X)P_E = x + (1 - 2x)P_E. \text{ (unipolar)}$$

$$x' = (1 - 2P_E)x. \text{ (bipolar)}$$

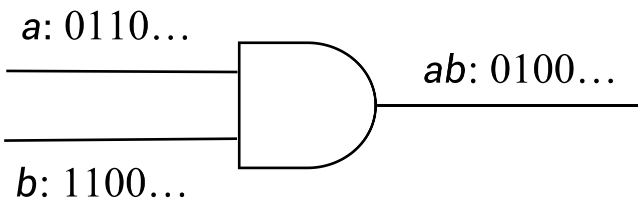
P_X → → P_X'

01000101...... → 01010101...... → 01010101......

↑ Each bit has a probability of P_E to flip

Bit flip models of stochastic multipliers

- Unipolar stochastic multipliers

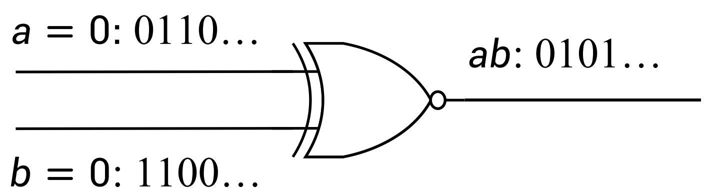


$$a'b' = (1-2P_E)^2ab + (1-2P_E)P_E(a+b) + P_E^2.$$

- Error recovery

Given $a'b'$, cannot directly compute ab

- Bipolar stochastic multipliers



$$a'b' = (1-2P_E)^2ab$$

- Error recovery

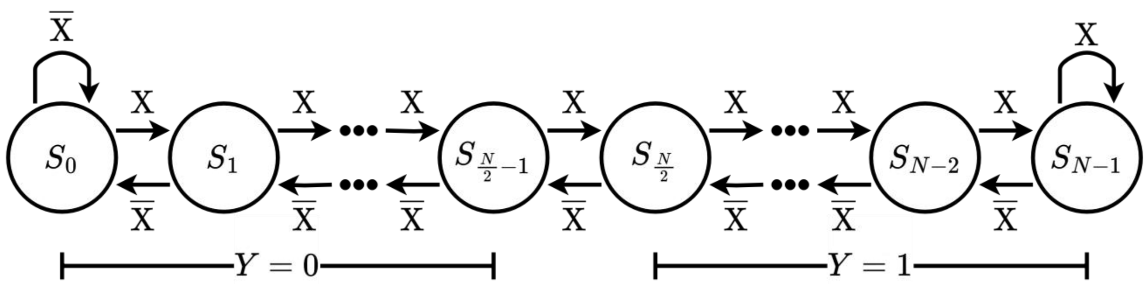
Given $a'b'$, directly compute ab by a linear transformation

$$Recovery\ value = \frac{Distorted\ value}{(1-2P_E)^2}$$

Not possible in conventional arithmetic!

Bit flip models of stochastic FSM

- FSM-based stochastic circuits, Stanh as an example



Stochastic FSM implementing tanh function

$$y' = Stanh[\frac{N}{2}(1-2P_E)x]$$

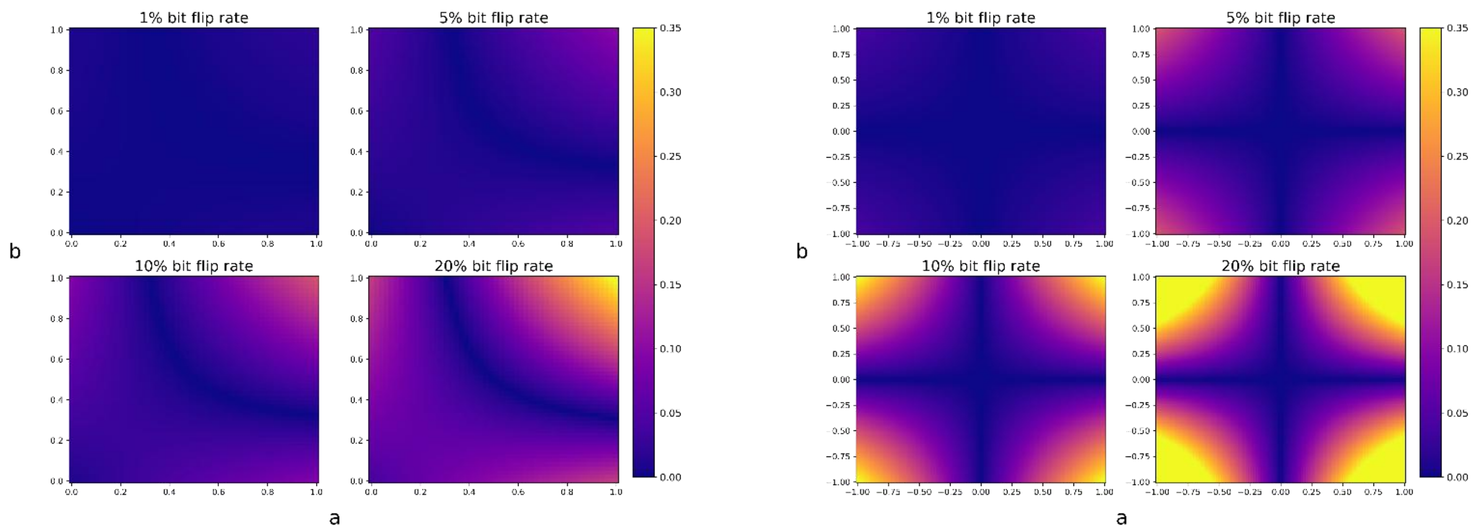
- Error recovery

$$y = \frac{\frac{1+y'}{1-y'}^{\frac{1}{1-2P_E}} - 1}{\frac{1+y'}{1-y'}^{\frac{1}{1-2P_E}} + 1}$$

Not possible in conventional arithmetic!

Experiments & Results

- Verification of the bit-flip models for stochastic multipliers



Absolute error of stochastic multiplication with different bit flip rates in the (left) unipolar representation and (right) bipolar representation.

Error is 0 when

$$a = b = 0.5 \; or \; \frac{a}{2a-1} + \frac{b}{2b-1} = P_E \; (a \neq 0.5, \; b \neq 0.5)$$

Error is 0 when

$$a = 0 \; or \; b = 0$$

Error recovery of bipolar stochastic multipliers

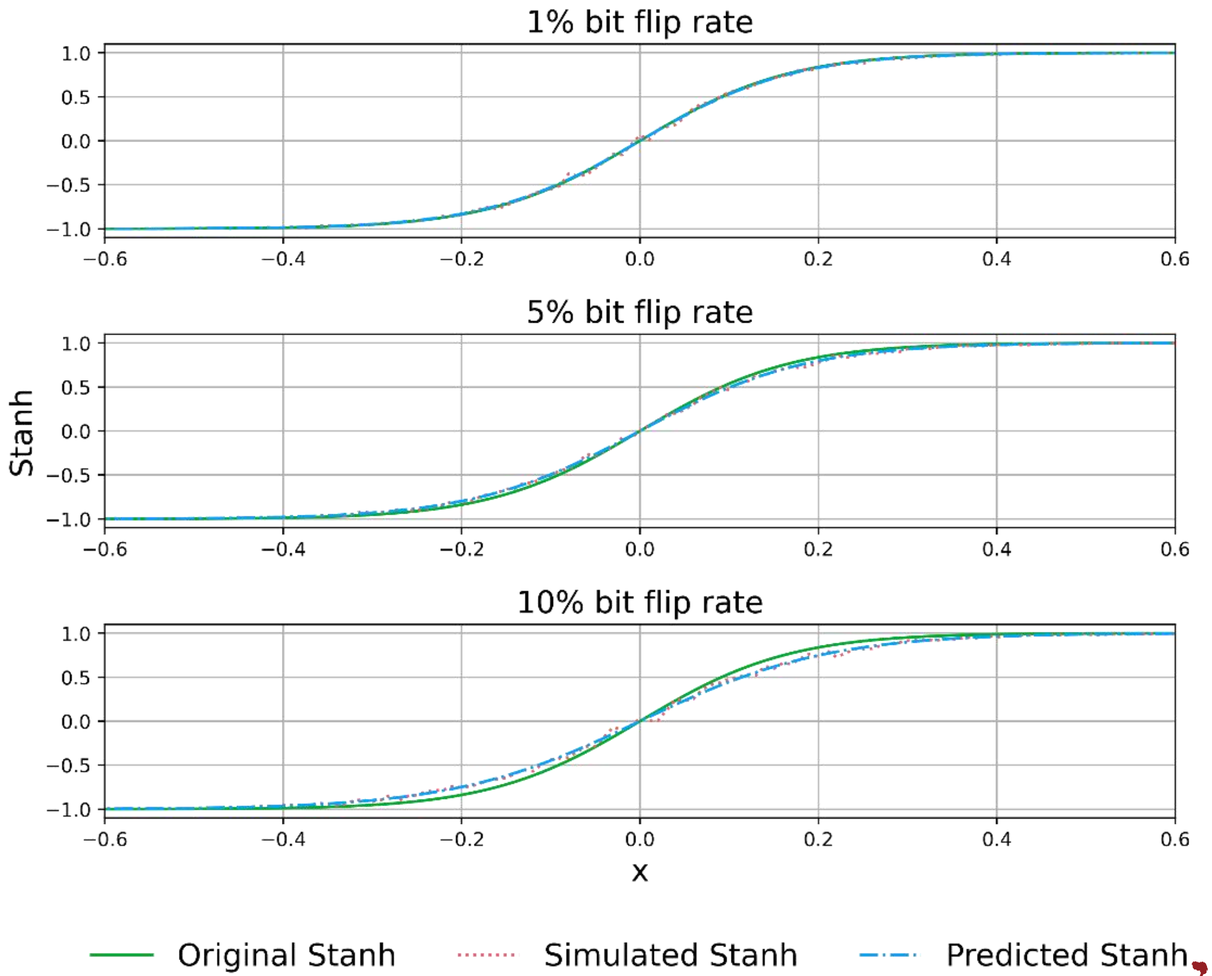
- Error recovery with different bit flip rates.
- 100 trials are performed, and RMSEs are measured.

Table 1. the RMSEs of stochasitc mulitpliers before and after error recovery

Bit-flip rate	0%	1%	5%	10%
Distorted	0.0011	0.0164	0.0514	0.0941
Recovered	N/A	0.0011	0.0015	0.0021

Stothcastic FSM models

- Verification of the bit-flip model



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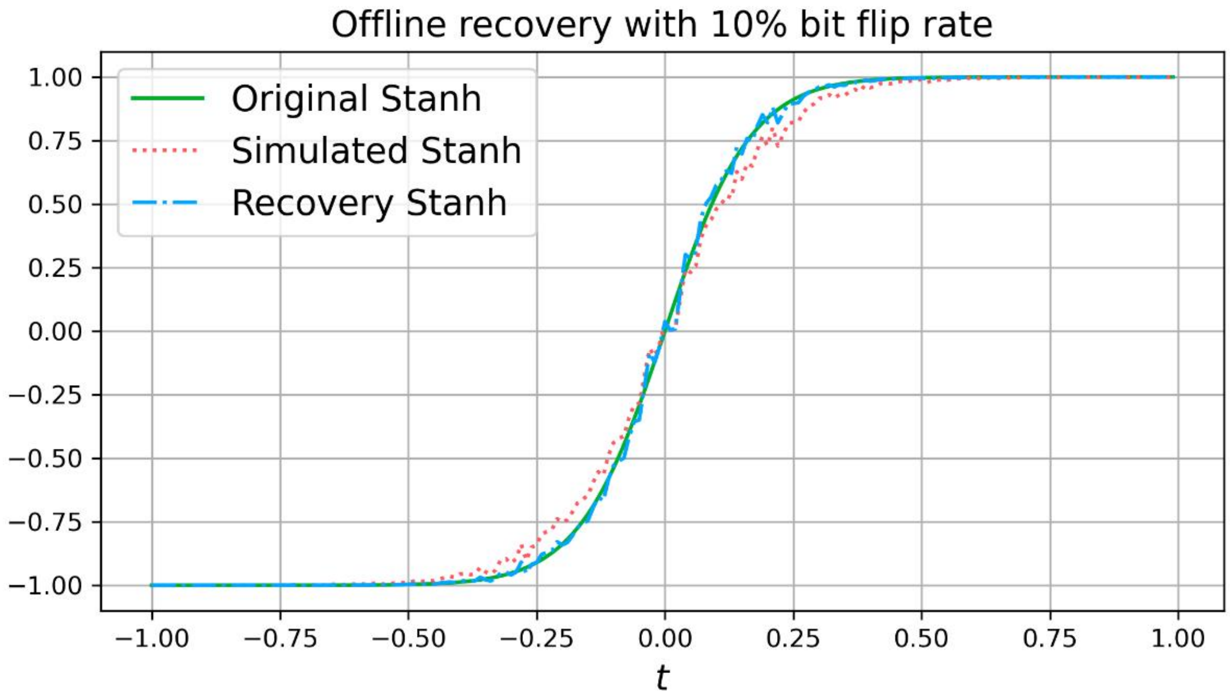
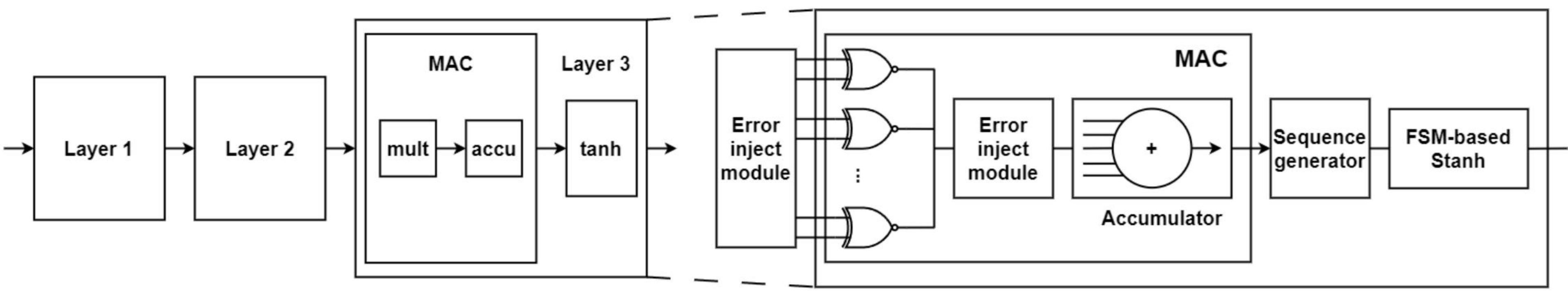
Stothcastic FSM models (Cont'd)

- Error recovery



Application in a multi-layer perceptron (MLP)

- A three-layer (784-256-256-10) MLP is implemented by the aforementioned SC components
- For each layer: $f_i(x_i) = \tanh(2(x_i \times w_i))$,
- Equation for the entire network: $output = f_3(f_2(f_1(x)))$.



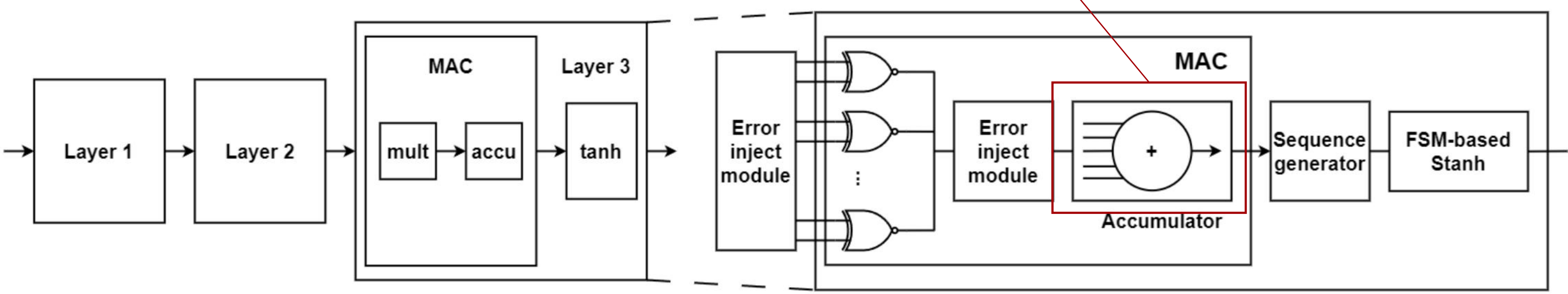
(a) Error-free SC-based MLP circuit

(b) SC-based MLP circuit with error injection

Error model of the SC accumulator

$$\mathbb{E}[sum] = \sum(2P_{X_k} - 1) = \sum x_k$$

$$sum^d = \sum x' = \sum(1 - 2P_E)x_k$$
$$= (1 - 2P_E)sum$$



(a) Error-free SC-based MLP circuit

(b) SC-based MLP circuit with error injection

Error model of the entire MLP circuit

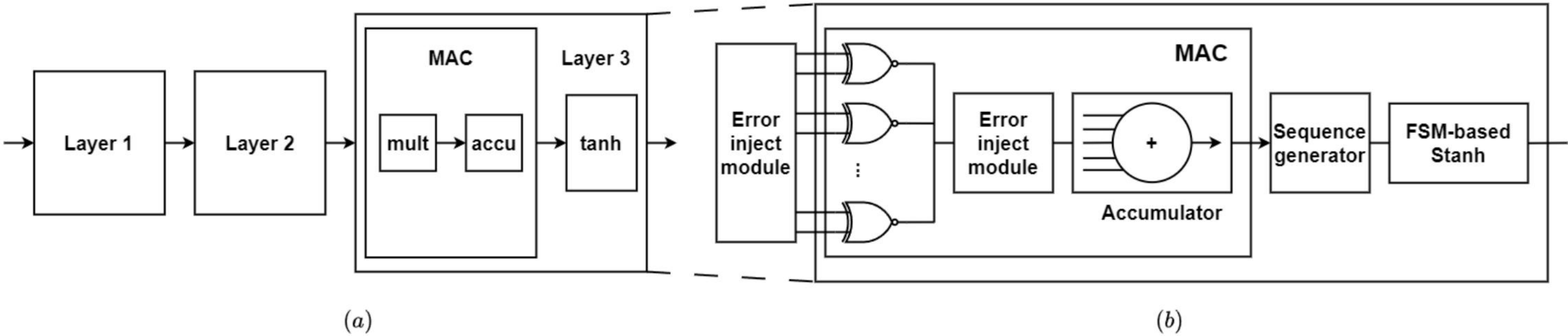
$$f_i(x_i) = \tanh(2(x_i \times w_i))$$

$$output = f_3(f_2(f_1(x)))$$

$$\Rightarrow$$

$$f_i^d(x_i) = \tanh(2(1-2P_E)^4(x_i \times w_i))$$

$$output^d = f_3^d(f_2^d(f_1^d(x)))$$



Error-free SC-based MLP circuit

SC-based MLP circuit with error injection

Verification of the bit flip model

- A three-layer (784-256-256-10) MLP is implemented by the aforementioned SC components on MNIST dataset.

Table 2: Accuracies of simulated and predicted results under different bit-flip rates

Model	Bit flip rate (%)	Simulated accuracy (%)	Predicted accuracy (%)
Pre-trained MLP	0	97.77	-
SC-MLP	0	97.71	97.77
	1	97.33	97.75
	5	96.92	97.48
	10	94.84	96.65

Error recovery of the SC-based MLP circuit

- An online layer-by-layer error recovery strategy is proposed
- For each layer:

$$f_i(x) = \frac{\dfrac{1 + f_i^d(x)}{1 - f_i^d(x)}^{\frac{1}{(1-2P_E)^4}} - 1}{\dfrac{1 + f_i^d(x)}{1 - f_i^d(x)}^{\frac{1}{(1-2P_E)^4}} + 1}$$

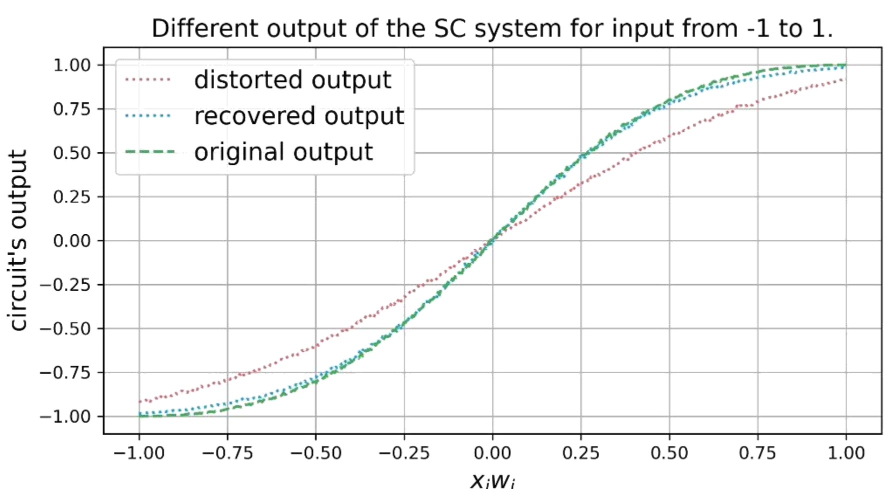
$$f_i(x_i) = \tanh(2(x_i \times w_i))$$

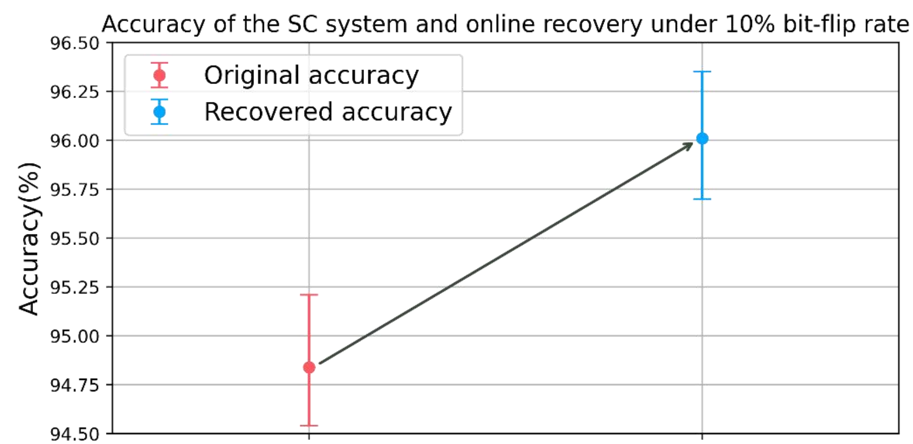
$$output = f_3(f_2(f_1(x)))$$

$$\Leftarrow$$

$$f_i^d(x_i) = \tanh(2(1 - 2P_E)^4(x_i \times w_i))$$

$$output^d = f_3^d(f_2^d(f_1^d(x)))$$





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Conclusion and future work

- Bit-flip models of stochastic sequences and multiple stochastic circuits are established;
- Formulation are conducted and bit-flip error recovery is proved to be possible on bipolar stochastic arithmetics;
- Bit-flip error recovery on independent SC components shows a high accuracy;
- An online layer-by-layer error recovery strategy is proposed to deal with an SC-based MLP circuit. The accuracy is partially recovered.
- More factors will be considered in the future work to build more accurate SC bit-flip models.

Thank you for your attention!

Q&A