Joint Transmit Waveforms and Receive Filters Design for Large-Scale MIMO Beampattern Synthesis

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***Abstract*—In this paper, the joint design of transmit waveforms and receive filters for multiple-input multiple-output (MIMO) beampattern synthesis problem is considered. This problem is formulated to approximate a desired transmit beampattern (i.e., an energy distribution in space and frequency) and to minimize the cross-correlation of waveform signals between different spatial angles by considering various practical waveform constraints. The optimization problem is highly nonconvex due to the nonconvexity of the objective function as well as the waveform constraints. To solve this problem, an efficient algorithm is proposed based on the block majorization-minimization (BMM) method. Performance of the proposed algorithm compared to the state-of-the-art is demonstrated through numerical simulations.**

***Index Terms*—Multiple-input multiple-output (MIMO) system, waveform diversity, mismatched filter bank, beampattern design, nonconvex optimization.**

I. Introduction

Multiple-input multiple-output (MIMO) systems [1] have the capacity to transmit independent probing signal or waveforms from each transmit antenna. Such waveform diversity feature leads to numerous desirable properties for MIMO systems. For example, a modern MIMO radar system, compared to the classical phased-array radar, has appealing features like higher spatial resolution [2], superior moving-target detection [3], and better parameter identifiability [4], [5].

The MIMO beampattern matching problem is critically important in many scenarios, like in defense systems, communication systems, and biomedical applications. This problem is concerned with designing the probing waveforms to approximate a desired antenna array transmit beampattern (i.e., an energy distribution in space and frequency) and to minimize the cross-correlation of the signals reflected back from various targets of interest by considering some practical waveform constraints which represent desirable properties and/or are enforced from a hardware implementation perspective [6]. The MIMO transmit beampattern matching problem appears to be difficult from an optimization point of view due to the existence of the fourth-order nonconvex objective function and the possibly nonconvex waveform constraints.

In [7], the MIMO transmit beampattern matching problem was formulated to minimize the difference between the designed beampattern and a desired one. The aforementioned problem was further modified in [8], [9] by introducing the cross-correlation between the signals. In [9], the authors proposed to design the waveform covariance matrix to match the desired beampattern through semidefinite programming. A closed-form waveform covariance matrix design method was also proposed based on discrete Fourier transform (DFT) coefficients and Toeplitz matrices in [10], [11]. But such kind of methods can perform badly for small number of antennas. After the waveform covariance matrix is obtained, other methods should be applied to synthesize a desired waveform from its covariance matrix. For example, a cyclic algorithm was proposed in [12] to synthesize a constant modulus waveform from its covariance matrix. These methods are usually called two-stage methods. In practice, they could be inefficient and suboptimal if multiple waveform constraints are considered.

In [13], it was found that directly designing the waveform to match the desired beampattern can give a better performance, which is referred to as the one-stage method. To facilitate efficient computation, authors in [14] introduce a receive filter bank and separate the optimization of transmit waveforms and receive filter bank (not necessarily matched to the transmit waveform as in the previous works). An block coordinate descent algorithm is proposed where the optimization of the receive filter bank is a convex least squares problem. The transmit waveform optimization is a norm-constrained nonconvex least squares problem which is solved by a low rank SDP relaxation method. The proposed algorithm in [14] is computationally expensive since an SDP should be solved in every iteration. Moreover, the SDP-based algorithm is designed by leveraging on the specific structure of the unimodulus constraint [14], and however many other practical waveform constraints should be considered in practical design.

The block majorization-minimization (BMM) method [15], [16] has shown its great efficiency in deriving fast and convergent algorithms to solve nonconvex constrained problems [17], [18]. In this paper, we propose to solve the MIMO transmit beampattern matching problem based on the BMM method by considering different waveform constraints. By alternating between the optimization of the transmit waveforms and the receive filter bank, we are able to achieve a high degree of suppression of cross-correlation levels between different spatial angles, while closely approximating a desired beampattern. The proposed BMM-based algorithm is compared with the existing one through numerical simulations.

II. MIMO Beampattern Matching Problem Formulation

A colocated MIMO radar [19] with M transmit antennas in a uniform linear array (ULA)[1], as shown in Fig.

[1]It should be noted that the design techniques proposed in this paper can also be extended some more general array settings like the 2D planar array.

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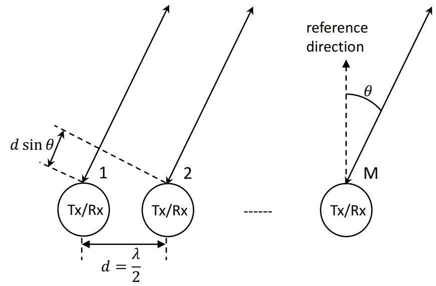

Figure 1. MIMO transceiver with M antennas (θ: spacial direction of interest).

1, is considered. Each transmit antenna can emit a different waveform $x_m(n) \in \mathbb{C}$ with $m = 1, 2, \ldots, M$, $n = 1, 2, \ldots, N$, where N is the number of samples. Let $\mathbf{x}(n) = [x_1(n), x_2(n), \ldots, x_M(n)]^T$ be the nth sample of the M transmit waveforms. We denote the waveform vector $\mathbf{x} = \left[\mathbf{x}^T(1), \mathbf{x}^T(2), \ldots, \mathbf{x}^T(N)\right]^T \in \mathbb{C}^{MN}$.

The signal at a target location with angle $\theta \in \Theta$ (the angle set covering the direction of interest) can be represented by $\sum_{m=1}^{M} e^{-j\pi(m-1)\sin\theta} x_m(n) = \mathbf{a}^T(\theta)\mathbf{x}(n),\ n = 1, \ldots, N,$ where $\mathbf{a}(\theta)$ is the transmit steering vector written as $\mathbf{a}(\theta) = \left[1, e^{-j\pi\sin\theta}, \ldots, e^{-j\pi(M-1)\sin\theta}\right]^T$. Then, the power for the probing signal $\mathbf{x}$ from the transmitter at location θ which is named the *transmit beampattern* is given by

$$\begin{aligned} P_t(\theta, \mathbf{x}) &= \textstyle\sum_{n=1}^{N} \left(\mathbf{a}^T(\theta)\mathbf{x}(n)\right)^* \left(\mathbf{a}^T(\theta)\mathbf{x}(n)\right) \\ &= \left(\left(\mathbf{I}_N \otimes \mathbf{a}^T(\theta)\right)\mathbf{x}\right)^H \left(\left(\mathbf{I}_N \otimes \mathbf{a}^T(\theta)\right)\mathbf{x}\right) \\ &= \mathbf{x}^H \left(\mathbf{I}_N \otimes \mathbf{a}^*(\theta)\mathbf{a}^T(\theta)\right)\mathbf{x} = \mathbf{x}^H \mathbf{A}(\theta)\mathbf{x}, \end{aligned}$$

where $\mathbf{A}(\theta) \triangleq \mathbf{I}_N \otimes \mathbf{a}^*(\theta)\mathbf{a}^T(\theta)$. The concept of beampattern can also be extended to the receiver side which is named as the *receive beampattern* given by

$$\begin{aligned} P_r(\theta, \mathbf{x}, \mathbf{h}) &= \textstyle\sum_{n=1}^{N} \left(\mathbf{a}^T(\theta)\mathbf{h}(n)\right)^* \left(\mathbf{a}^T(\theta)\mathbf{x}(n)\right) \\ &= \mathbf{h}^H \left(\mathbf{I}_N \otimes \mathbf{a}^*(\theta)\mathbf{a}^T(\theta)\right)\mathbf{x} = \mathbf{h}^H \mathbf{A}(\theta)\mathbf{x}, \end{aligned}$$

where $\mathbf{h} \in \mathbb{C}^{MN}$ is the filter bank at the receiver. In particular, when matched filters are employed in the system, i.e., $\mathbf{h} = \mathbf{x}$, the transmit beampattern and receive beampattern coincide.

In applications like MIMO radars, the transmitted waveforms probing to different directions are required to hold good autocorrelation property. This is because MIMO radar has the ability to form multiple spatial beams simultaneously and the target echoed signal or clutter from different beams will be interference to each other. (Notice that clutters have different spatial directions with respect to useful echoes.) Therefore, in order to improve the detection performance of the small target close to a large target and to avoid false alarm introduced by clutter, it is ideal that the transmitted waveforms for different spatial directions have small enough correlation coefficients. Suppose there are $\overline{K}$ targets of interest, and then the spatial cross-correlation sidelobes between the probing signals at locations θ_i and θ_j ($i \neq j$, $i, j = 1, \ldots, \overline{K}$, and $\theta_i, \theta_j \in \Theta$) is

$$\begin{aligned} P_{cc}(\theta_i, \theta_j, \mathbf{x}, \mathbf{h}) &= \textstyle\sum_{n=1}^{N} \left(\mathbf{a}^T(\theta_i)\mathbf{h}(n)\right)^* \left(\mathbf{a}^T(\theta_j)\mathbf{x}(n)\right) \\ &= \left(\left(\mathbf{I}_N \otimes \mathbf{a}^T(\theta_i)\right)\mathbf{h}\right)^H \left(\left(\mathbf{I}_N \otimes \mathbf{a}^T(\theta_j)\right)\mathbf{x}\right) \\ &= \mathbf{h}^H \left(\mathbf{I}_N \otimes \mathbf{a}^*(\theta_i)\mathbf{a}^T(\theta_j)\right)\mathbf{x} = \mathbf{h}^H \mathbf{A}(\theta_i, \theta_j)\mathbf{x}, \end{aligned}$$

where $\mathbf{A}(\theta_i, \theta_j) \triangleq \mathbf{I}_N \otimes \mathbf{a}^*(\theta_i)\mathbf{a}^T(\theta_j)$.

Finally, the considered two objectives of the transmit beampattern matching problem are as follows: i) to match a desired transmit beampattern denoted as $p(\theta)$, which is given by[2]

$$J(\alpha, \mathbf{x}, \mathbf{h}) = \textstyle\sum_{\theta\in\Theta} \omega(\theta) \left|\alpha p(\theta) - P_r(\theta, \mathbf{x}, \mathbf{h})\right|^2, \quad (1)$$

where $\omega(\theta) \geq 0$ is the weight for the direction θ; and ii) to minimize the cross-correlation between the probing signals at a number of given target locations due to the fact that the statistical performance of adaptive MIMO radar techniques rely on the cross-correlation beampattern, which is given as

$$E(\mathbf{x}, \mathbf{h}) = \textstyle\sum_{\theta_i, \theta_j\in\Theta,\, i\neq j} \left|P_{cc}(\theta_i, \theta_j, \mathbf{x}, \mathbf{h})\right|^2. \quad (2)$$

By considering $J(\alpha, \mathbf{x}, \mathbf{h})$ and $E(\mathbf{x}, \mathbf{h})$, the MIMO transmit beampattern matching problem is formulated as follows:

$$\begin{aligned} \underset{\alpha, \mathbf{x}, \mathbf{h}}{\text{minimize}} \quad & F(\alpha, \mathbf{x}, \mathbf{h}) \triangleq J(\alpha, \mathbf{x}, \mathbf{h}) + \omega_{cc} E(\mathbf{x}, \mathbf{h}) \\ \text{subject to} \quad & \mathbf{x} \in \mathcal{X} \triangleq \mathcal{X}_0 \cap (\cap_i \mathcal{X}_i), \end{aligned} \quad (3)$$

where ω_{cc} controls the sidelobe term and $\mathcal{X}$ generally denotes the waveform constraint with $\mathcal{X}_0 \triangleq \left\{\mathbf{x} \mid \sum_{n=1}^{N} |x_m(n)|^2 = c_e^2\right\}$ representing the *total transmit energy constraint*. Some other practical waveform constraints are also considered. *The constant modulus constraint* $\mathcal{X}_1 \triangleq \left\{\mathbf{x} \mid |x(l)| = c_d = \frac{c_e}{\sqrt{N}}\right\}$ for $l = 1, \ldots, MN$ is to prevent the non-linearity distortion of the power amplifier to maximize the efficiency of the transmitter. The peak-to-average ratio $\mathrm{PAR}(\mathbf{x}_m) = \frac{\max|x_m(n)|^2}{\sum_n |x_m(n)|^2/N}$ represents the peak signal power to its average power that is constrained to a small threshold, so that the analog-to-digital and digital-to-analog converters can have lower dynamic range and fewer linear power amplifiers are needed. Since $\mathcal{X}_0$ exists, the *PAR constraint* becomes $\mathcal{X}_2 \triangleq \{\mathbf{x} \mid |x(l)| \leq c_p\}$. *The similarity constraint* $\mathcal{X}_3 \triangleq \{\mathbf{x} \mid |\mathbf{x} - \mathbf{x}_{\mathrm{ref}}| \leq c_\epsilon\}$ is to allow the designed waveforms to lie in the neighborhood of a reference one $\mathbf{x}_{\mathrm{ref}}$ which already attains a good performance [20].

Problem (3) is a constrained nonconvex problem due to the nonconvex objective and constraints. We aims at solving this problem by using efficient nonconvex optimization method.

III. Problem Solving via The BMM Method

A. The Block Majorization-Minimization (BMM) Method

Block majorization-minimization (BMM) method [16] can be viewed as a judicious combination of the renowned block coordinate descent (BCD) method [21] and the majorization-minimization (MM) method [17]. The BCD method aims to find a local optimum of an objective function by optimizing it along one variable block at a time. Instead of solving for an exact update, MM resorts to minimize the potentially complicated objective function by iteratively solving an upperbound function through carrying out an inexact variable update. As a combination of the two methods, BMM aims at minimizing an objective by applying MM to different blocks cyclically. Suppose an optimization problem is given as

$$\underset{\mathbf{x}}{\text{minimize}} \quad f(\mathbf{x}) \quad \text{subject to} \quad \mathbf{x} \in \mathcal{X},$$

[2]Variable α is introduced since $p(\theta)$ is typically given in a "normalized form" and we want to approximate a scaled version of $p(\theta)$, not $p(\theta)$ itself. The subscript t in $P_r(\theta, \mathbf{x}, \mathbf{h})$ is omitted henceforth.

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where $\mathcal{X} \subseteq \mathbb{R}^N$. Suppose the optimization variable $\mathbf{x}$ can be partitioned into I blocks as $\mathbf{x} \triangleq (\mathbf{x}_1, \ldots, \mathbf{x}_I)$ where $\mathbf{x}_i \in \mathcal{X}_i$ and $\mathcal{X} = \prod_{i=1}^{I} \mathcal{X}_i$ with $\mathcal{X}_i \subseteq \mathbb{R}^{n_i}$ and $\sum_{i=1}^{I} n_i = N$. Instead of dealing with the original problem, the BMM method solves a series of simple surrogate subproblems.

Specifically, at each iteration of BMM method, the variable block $\mathbf{x}_i$ is updated according to the following update rules:

$$\begin{cases} \mathbf{x}_i^{(t)} \in \arg\min_{\mathbf{x}_i \in \mathcal{X}_i} \overline{f}_i(\mathbf{x}_i, \mathbf{x}^{(t-1)}), \\ \mathbf{x}_{-i}^{(t)} = \mathbf{x}_{-i}^{(t-1)}, \text{ with } \mathbf{x}_{-i} \triangleq (\mathbf{x}_1, \ldots, \mathbf{x}_{i-1}, \mathbf{x}_{i+1}, \ldots, \mathbf{x}_I), \end{cases}$$

where $\overline{f_i}$ is a majorizing function for f with respect to $\mathbf{x}_i$. The algorithm iteratively runs until some convergence criterion is met. To claim convergence for BMM with convex $\mathcal{X}_i$, the surrogate function $\overline{f_i}$ needs to satisfy the following assumptions: **A1)** $\overline{f_i}(\mathbf{x}_i^{(t)}; \mathbf{x}^{(t)}) = f(\mathbf{x}^{(t)})$ with $\forall \mathbf{x}_i^{(t)} \in \mathcal{X}_i$; **A2)** $\overline{f_i}(\mathbf{x}_i; \mathbf{x}^{(t)}) \geq f(\mathbf{x}_1^{(t)}, \ldots, \mathbf{x}_i, \ldots, \mathbf{x}_m^{(t)})$ with $\forall \mathbf{x}_i \in \mathcal{X}_i$, $\forall \mathbf{x}^{(t)} \in \mathcal{X}$; **A3)** $\nabla_{\mathbf{x}_i} \overline{f_i}(\mathbf{x}_i^{(t)}; \mathbf{x}^{(t)}, \mathbf{d}_i) = \nabla f(\mathbf{x}^{(t)}; \mathbf{d}_i^0)$, $\forall \mathbf{d}_i^0$ s.t. $\mathbf{x}_i^{(t)} + \mathbf{d}_i \in \mathcal{X}_i$ and $\mathbf{d}_i^0 \triangleq (\mathbf{0}, \ldots, \mathbf{d}_i, \ldots, \mathbf{0})$. For a nonconvex set $\mathcal{X}_i$, to claim stationarity convergence, the **A3)** above should be modified as **A3')** $\nabla_{\mathbf{x}_i} \overline{f}_i \left(\mathbf{x}_i^{(t)}; \mathbf{x}_i^{(t)}, \mathbf{d}_i\right) = \nabla f\left(\mathbf{x}^{(t)}; \mathbf{d}_i^0\right)$, $\forall \mathbf{d}_i^0 \in \mathcal{T}_{\mathcal{X}_i}\left(\mathbf{x}^{(t)}\right)$ and $\mathbf{d}_i^0 \triangleq (\mathbf{0}, \ldots, \mathbf{d}_i, \ldots, \mathbf{0})$.

Using BMM for MIMO transmit beampattern design has been explored in [22], [23], [24], in this paper, we show how to extend the BMM method to the joint design of transmit waveforms and receive filters in MIMO systems. In the following, we discuss the problem solving procedure with respect to each variable block.

B. Solving for The Scaling Parameter (α-Subproblem)

The variable α is only involved in the beampattern matching term $J(\alpha, \mathbf{x}, \mathbf{h})$ as follows:

$$\begin{aligned} J(\alpha, \mathbf{x}, \mathbf{h}) &= \textstyle\sum_{\theta \in \Theta} \omega(\theta) \mathbf{x}^H \mathbf{A}(\theta) \mathbf{h}\mathbf{h}^H \mathbf{A}(\theta) \mathbf{x} \\ &-2\alpha \mathrm{Re}\left\{\textstyle\sum_{\theta \in \Theta} \omega(\theta) p(\theta) \mathbf{h}^H \mathbf{A}(\theta) \mathbf{x}\right\} \\ &+\alpha^2 \textstyle\sum_{\theta \in \Theta} \omega(\theta) (p(\theta))^2, \end{aligned} \tag{4}$$

which is convex and quadratic in α. Then, given the current iterate $\left\{\alpha^{(t)}, \mathbf{x}^{(t)}, \mathbf{h}^{(t)}\right\}$, it readily follows the minimum of $F(\alpha, \mathbf{x}, \mathbf{h})$ is attained when

$$\alpha\left(\alpha^{(t)}, \mathbf{x}^{(t)}, \mathbf{h}^{(t)}\right) = \frac{\sum_{\theta \in \Theta} \omega(\theta) p(\theta) P_r\left(\theta, \mathbf{x}^{(t)}, \mathbf{h}^{(t)}\right)}{\sum_{\theta \in \Theta} \omega(\theta) (p(\theta))^2}. \tag{5}$$

C. Solving for The Transmit Waveforms ($\mathbf{x}$-Subproblem)

The objective $F(\alpha, \mathbf{x}, \mathbf{h})$ is quadratic and convex in $\mathbf{x}$ (both $P(\theta, \mathbf{x}, \mathbf{h})$ and $P_{cc}(\theta_i, \theta_j, \mathbf{x}, \mathbf{h})$ are linear in $\mathbf{x}$), but no closed-form solution can be directly derived for $\mathbf{x}$ due to the existence of the nonconvex waveform constraints $\mathcal{X}$[3]. In this section, we will discuss how to solve the $\mathbf{x}$-subproblem based on the majorization minimization techniques.

We first introduce the following lemma.

Lemma 1. *Given iterate* $\left\{\alpha^{(t)}, \mathbf{x}^{(t)}, \mathbf{h}^{(t)}\right\}$ *and defining* $\mathbf{M}_{J,x}^{(t)} \triangleq \sum_{\theta \in \Theta} \omega(\theta) \mathbf{A}(\theta) \mathbf{h}^{(t)} \mathbf{h}^{(t)H} \mathbf{A}(\theta)$, *we have*

$$\begin{aligned} \mathbf{x}^H \mathbf{M}_{J,x}^{(t)} \mathbf{x} \leq \mathbf{x}^{(t)H} \mathbf{M}_{J,x}^{(t)} \mathbf{x}^{(t)} + 2\mathrm{Re}\left\{\mathbf{x}^{(t)H} \mathbf{M}_{J,x}^{(t)} \left(\mathbf{x} - \mathbf{x}^{(t)}\right)\right\} \\ +\psi(\mathbf{M}_{J,x}^{(t)}) \left\|\mathbf{x} - \mathbf{x}^{(t)}\right\|_2^2, \end{aligned}$$

[3]It is a NP-hard unimodular quadratic program even only considering $\mathcal{X}_1$.

where the constant $\psi_J^{(t)}(\mathbf{M}_{J,x}) \geq \lambda_{\max}(\mathbf{M}_{J,x}^{(t)})$ *(the largest eigenvalue of* $\mathbf{M}_{J,x}^{(t)}$*).*

Based on Lemma 1, at iterate $\left\{\alpha^{(t+1)}, \mathbf{x}^{(t)}, \mathbf{h}^{(t)}\right\}$, we have the majorizing function for $J(\alpha, \mathbf{x}, \mathbf{h})$ as

$$\begin{aligned} &\overline{J}_x^{(t)}(\mathbf{x}) = \psi(\mathbf{M}_{J,x}^{(t)}) \|\mathbf{x}\|_2^2 \\ &+2\mathrm{Re}\{[\textstyle\sum_{\theta \in \Theta} \omega(\theta) (P(\theta, \mathbf{x}^{(t)}, \mathbf{h}^{(t)})^H \\ &-\alpha^{(t+1)} p(\theta)) \mathbf{h}^{(t)H} \mathbf{A}(\theta) - \psi(\mathbf{M}_{J,x}^{(t)}) \mathbf{x}^{(t)H}] \mathbf{x}\} + const., \end{aligned}$$

where the first term is a constant since $\mathbf{x}^H \mathbf{x} = \|\mathbf{x}\|_2^2 = Mc_e^2$. Then we get the following majorizing function

$$\overline{J}_x^{(t)}(\mathbf{x}) = 2\mathrm{Re}\{\mathbf{y}_{J,x}^{(t)H} \mathbf{x}\} + const.,$$

where $\mathbf{y}_{J,x}^{(t)} \triangleq \left[\sum_{\theta \in \Theta} \omega(\theta) \left(P\left(\theta, \mathbf{x}^{(t)}, \mathbf{h}^{(t)}\right)^H - \alpha^{(t+1)} p(\theta)\right) \mathbf{A}(\theta) \mathbf{h}^{(t)} - \psi(\mathbf{M}_{J,x}^{(t)}) \mathbf{x}^{(t)}\right]$.

To deal with the sidelobe suppression term $E(\mathbf{x}, \mathbf{h})$ in Problem (3), similar majorization steps based on Lemma 1 as above will be applied, which is summarized in the following.

Lemma 2. *[22] Given* $\left(\alpha^{(t+1)}, \mathbf{x}^{(t)}, \mathbf{h}^{(t)}\right)$, $E(\mathbf{x}, \mathbf{h})$ *is linearly majorized by the following function*

$$\overline{E}_x^{(t)}(\mathbf{x}) = 2\mathrm{Re}\{\mathbf{y}_{E,x}^{(t)H} \mathbf{x}\} + const.,$$

where $\mathbf{y}_{E,x}^{(t)} \triangleq \sum_{\theta_i, \theta_j \in \overline{\Theta}} [(P_{cc}(\theta_i, \theta_j, \mathbf{x}^{(t)}, \mathbf{h}^{(t)}))^H \mathbf{A}(\theta_i, \theta_j) \mathbf{h}^{(t)} - \psi(\mathbf{M}_{E,x}^{(t)}) \mathbf{x}^{(t)}]$ *with* $\mathbf{M}_{E,x}^{(t)} \triangleq \sum_{\theta_i, \theta_j \in \overline{\Theta}} \mathbf{A}(\theta_j, \theta_i) \mathbf{h}^{(t)} \mathbf{h}^{(t)H} \mathbf{A}(\theta_i, \theta_j)$.

Finally, by combing the above two majorizing functions $\overline{J}_x^{(t)}(\mathbf{x})$ and $\overline{E}_x^{(t)}(\mathbf{x})$, the overall majorizing function for $\overline{F}_x^{(t)}(\mathbf{x})$ is given as follows:

$$\overline{F}_x^{(t)}(\mathbf{x}) = \mathrm{Re}\left(\mathbf{y}_x^{(t)H} \mathbf{x}\right) + const.,$$

where $\mathbf{y}_x^{(t)} \triangleq 2\mathbf{y}_{J,x}^{(t)} + \omega_{cc} 2\mathbf{y}_{M,x}^{(t)}$. Finally, the $\mathbf{x}$-subproblem is given as follows:

$$\min_{\mathbf{x}} \quad \mathrm{Re}\left(\mathbf{y}_x^{(t)H} \mathbf{x}\right) \quad \text{s.t.} \quad \mathbf{x} \in \mathcal{X} \ . \tag{6}$$

For problem (6), as to different interested waveform constraints, closed-form optimal solutions $\mathbf{x}^\star$ can be derived, which are summarized in the following lemma.

Lemma 3. *Problem* (6) *can be solved distributedly over antennas and the solution* $\mathcal{M}(\mathbf{y}_x^{(t)})$ *with respect to different waveform constraints is given in [23], [25, Lemma 4].*

D. Solving for The Receive Filters ($\mathbf{h}$-Subproblem)

The objective $F(\alpha, \mathbf{x}, \mathbf{h})$ is quadratic and convex in $\mathbf{h}$ and then the $\mathbf{h}$-subproblem is a convex quadratic programming (c.f. Eq. (4)). Although a closed-form solution can be derived, when M and N are big, a large-dimension matrix inversion operation will be involved. To avoid this time-consuming procedure, we will try to make a easy computation by applying the majorization minimization method for $F(\alpha, \mathbf{x}, \mathbf{h})$. Again based on Lemma 1, we have the following result.

Lemma 4. *Given* $\left(\alpha^{(t+1)}, \mathbf{x}^{(t+1)}, \mathbf{h}^{(t)}\right)$, $J(\alpha, \mathbf{x}, \mathbf{h})$ *is quadratically majorized by the following functions*

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$$\overline{J}_h^{(t)}(\mathbf{h}) = \psi(\mathbf{M}_{J,h}^{(t)})\mathbf{h}^H\mathbf{h} - 2\mathrm{Re}\{\mathbf{y}_{J,h}^{(t)H}\mathbf{h}\} + const., \quad (7)$$

where $\psi(\mathbf{M}_{J,h}^{(t)}) \geq \lambda_{\max}(\mathbf{M}_{J,h}^{(t)})$ *with* $\mathbf{M}_{J,h}^{(t)} \triangleq \sum_{\theta\in\Theta}\omega(\theta)\,\mathbf{A}(\theta)\,\mathbf{x}^{(t+1)}\mathbf{x}^{(t+1)H}\mathbf{A}(\theta)$ *and* $\mathbf{y}_{J,h}^{(t)} \triangleq \sum_{\theta\in\Theta}\omega(\theta)\,(\alpha^{(t+1)}p(\theta) - P(\theta,\mathbf{x}^{(t+1)},\mathbf{h}^{(t)}))\mathbf{A}(\theta)\,\mathbf{x}^{(t+1)} + \psi(\mathbf{M}_{J,h}^{(t)})\mathbf{h}^{(t)}$. *Similarly,* $E(\mathbf{x},\mathbf{h})$ *is majorized by*

$$\overline{E}_h^{(t)}(\mathbf{h}) = \psi(\mathbf{M}_{E,h}^{(t)})\mathbf{h}^H\mathbf{h} - 2\mathrm{Re}\{\mathbf{y}_{E,h}^{(t)H}\mathbf{h}\} + const., \quad (8)$$

where $\psi(\mathbf{M}_{E,h}^{(t)}) \geq \lambda_{\max}(\mathbf{M}_{E,h}^{(t)})$ *with* $\mathbf{M}_{E,h}^{(t)} \triangleq \sum_{\theta_i,\theta_j\in\overline{\Theta}}\mathbf{A}(\theta_j,\theta_i)\,\mathbf{x}^{(t+1)}\mathbf{x}^{(t+1)H}\mathbf{A}(\theta_i,\theta_j)$ *and* $\mathbf{y}_{E,h}^{(t)} \triangleq \sum_{\theta_i,\theta_j\in\overline{\Theta}}((P_{cc}(\theta_i,\theta_j,\mathbf{x}^{(t+1)},\mathbf{h}^{(t)}))^H\mathbf{A}(\theta_i,\theta_j)\,\mathbf{x}^{(t+1)} - \psi(\mathbf{M}_{E,h}^{(t)})\mathbf{h}^{(t)})$.

Finally, the objective of the $\mathbf{h}$-subproblem is given by

$$\overline{F}_h^{(t)}(\mathbf{h}) = \|\mathbf{h} - \mathbf{y}_h^{(t)}\|_2^2 + const.,$$

where $\mathbf{y}_h^{(t)} \triangleq (\psi(\mathbf{M}_{J,h}^{(t)}) + \omega_{cc}\psi(\mathbf{M}_{E,h}^{(t)}))^{-1}(\mathbf{y}_{J,h}^{(t)} + \omega_{cc}\mathbf{y}_{E,h}^{(t)})$ with the problem given as follows:

$$\mathsf{minimize}_{\mathbf{h}} \quad \|\mathbf{y}_h^{(t)} - \mathbf{h}\|_2^2 \ , \quad (9)$$

which obtains the closed-form solution as $\mathbf{h} = \mathbf{y}_h^{(t)}$.

Based on BMM, in order to solve the original problem (3), we just need to iteratively solve the subproblems with a (mostly) closed-form solution update for each variable. Such an algorithm is suitable for large-scale beampattern matching since it is free of costly computations can be solved in parallel.

Finally, the overall algorithm is summarized as follows.

Input: $\mathbf{a}(\theta)$, $p(\theta)$, $\alpha^{(0)}$, $\mathbf{x}^{(0)}$, $\mathbf{h}^{(0)}$, and $t=0$.
Repeat
1. Update $\alpha^{(t+1)}$ via Eq. (5);
2. Update $\mathbf{x}^{(t+1)} \leftarrow \mathcal{M}(\mathbf{y}_x^{(t)})$ via Lemma 3;
3. Update $\mathbf{h}^{(t+1)} \leftarrow \mathbf{y}_h^{(t)}$;
4. $t = t+1$;

Until $\{\alpha, \mathbf{x}, \mathbf{h}\}$ and $F(\alpha,\mathbf{x},\mathbf{h})$ converge.
Output: $\alpha^\star$, $\mathbf{x}^\star$, and $\mathbf{h}^\star$.

IV. Numerical Simulations

The performance of the proposed algorithm for MIMO transmit beampattern matching is evaluated by numerical simulations. A colocated MIMO radar system is considered with a ULA comprising $M = 32$ antennas with half-wavelength spacing between adjacent antennas. Without loss of generality, the total transmit power is set to $c_e^2 = 1$. Each transmit pulse has $N = 10$ samples. The range of angle is $\Theta = (-90°, 90°)$ with spacing $1°$ under which the weight $\omega(\theta) = 1$ for $\theta \in \Theta$, and $\omega_c = 0$. We consider a desired beampattern with three targets or mainlobes ($K = 3$) at $\theta_1 = -40°$, $\theta_2 = 0°$, $\theta_3 = 40°$, and each width of them is $\triangle\theta = 20°$. The desired beampattern is

$$p(\theta) = \begin{cases} 1, & \theta \in [\theta_k - \triangle\theta/2, \theta_k + \triangle\theta/2]\,,\ k = 1,2,\ldots,K \\ 0, & \text{otherwise.} \end{cases}$$

We first compare the convergence property of the objective function in terms of CPU running time for the beampattern matching problem under unimodulus waveform constraint by using the proposed BMM-based algorithm and the SDP-based algorithm [14]. As shown in Fig. 2, the BMM-based algorithm can have a monotonic convergence property with much faster convergence speed than the benchmark.

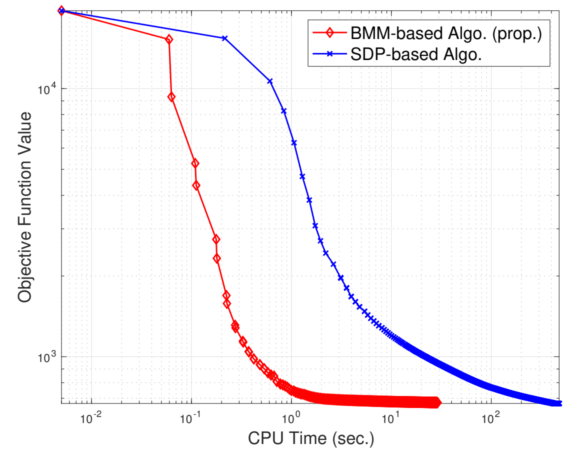

Figure 2. Convergence comparison for objective function value.

Then, we also compare the matching performance of the designed beampatterns in terms of the matching error defined as $\sum_{\theta\in\Theta}\omega(\theta)\,|\alpha p(\theta) - P(\theta,\mathbf{x},\mathbf{h})|^2$. The simulation results are reported in Fig. 3. We can see that compared to the benchmark, our proposed algorithm can have a tighter matching performance and can obtain a lower matching error, based on which the proposed algorithm is validated.

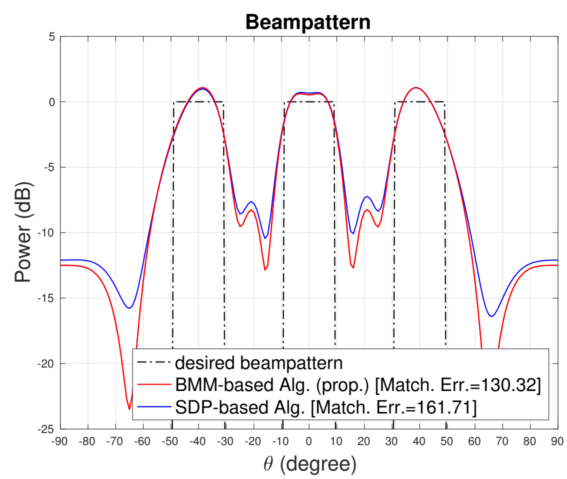

Figure 3. Transmit beampattern design with 3 targets .

V. Conclusions

This paper has considered the MIMO transmit beampattern matching problem via joint design of transceivers. Efficient algorithms have been proposed based on the block majorization minimization method. Numerical simulations show that the proposed algorithms are efficient in solving the beampattern matching problem and can obtain a better performance compared to the state-of-the-art.

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