MIMO TRANSMIT BEAMPATTERN MATCHING UNDER WAVEFORM CONSTRAINTS

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ABSTRACT

In this paper, the multiple-input multiple-output (MIMO) transmit beampattern matching problem is considered. The problem is formulated to approximate a desired transmit beampattern (i.e., an energy distribution in space and frequency) and to minimize the cross-correlation of signals reflected back to the array by considering different practical waveform constraints at the same time. Due to the nonconvexity of the objective function and the waveform constraints, the optimization problem is highly nonconvex. An efficient one-step method is proposed to solve this problem based on the majorization-minimization (MM) method. The performance of the proposed algorithms compared to the state-of-art algorithms is shown through numerical simulations.

***Index Terms*—** MIMO, waveform diversity, beampattern design, waveform constraints, nonconvex optimization.

1. INTRODUCTION

Multiple-input multiple-output (MIMO) systems [1] have the capacity to transmit independent probing signal or waveforms from each transmit antenna. Such waveform diversity feature leads to many desirable properties for MIMO systems. For example, a modern MIMO radar has many appealing features, like higher spatial resolution, superior moving target detection and better parameter identifiability, compared to the classical phased-array radar [2, 3, 4].

The MIMO transmit beampattern matching problem is critically important in many fields, like in defense systems, communication systems, and biomedical applications. This problem is concerned with designing the probing waveforms to approximate a desired antenna array transmit beampattern (i.e., an energy distribution in space and frequency) and also to minimize the the cross-correlation of the signals reflected back from various targets of interest by considering some practical waveform constraints. The MIMO transmit beampattern matching problem appears to be difficult from an optimization point of view because the existence of the fourth-order nonconvex objective function and the possibly nonconvex waveform constraints which are used to represent desirable properties and/or enforced from an hardware implementation perspective [5].

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In [6], the MIMO transmit beampattern matching problem was formulated to minimize the difference between the designed beampattern and the desired one. The formulation in [6] was modified in [7, 8] by introducing the cross-correlation between the signals. And in [8], the authors proposed to design the waveform covariance matrix to match the desired beampattern through semidefinite programming. A closed-form waveform covariance matrix design method was also proposed based on discrete Fourier transform (DFT) coefficients and Toeplitz matrices in [9, 10]. But such kind of methods can perform badly for small number of antennas. After the waveform covariance matrix is obtained, other methods should be applied to synthesize a desired waveform from its covariance matrix. For example, a cyclic algorithm was proposed in [11] to synthesize a constant modulus waveform from its covariance matrix. These methods are usually called two-steps methods. In practice, they could become inefficient and suboptimal if more waveform constraints are considered.

In [12], it was found that directly designing the waveform to match the desired beampattern can give a better performance, which is referred to as the one-step method. But the method in [12] is tailored to the constant modulus constraint and can be slow in convergence. In [13], the problem was solved based on the alternating direction method of multipliers (ADMM) [14]. However, again the proposed algorithm is only designed for dealing with unimodulus constraint.

The majorization-minimization (MM) method [15, 16] has shown its great efficiency in deriving fast and convergent algorithms to solve nonconvex problems in many different applications [17, 18]. In this paper, we propose a one-step method to directly solve the MIMO transmit beampattern matching problem based on the MM method by considering different waveform constraints. The performance of our algorithms compared to the existing algorithms is verified through numerical simulations.

2. MIMO TRANSMIT BEAMPATTERN MATCHING PROBLEM FORMULATION

A colocated MIMO radar [19] with M transmit antennas in a uniform linear array (ULA), as shown in Fig. 1, is considered. Each transmit antenna can emit a different waveform $x_m(n)$ with $m = 1, 2, \ldots, M$, $n = 1, 2, \ldots, N$, where N is the number of samples. Let $\mathbf{x}(n) = [x_1(n), x_2(n), \ldots, x_M(n)]^T$ be the nth sample of the M transmit waveforms and $\mathbf{x} =$

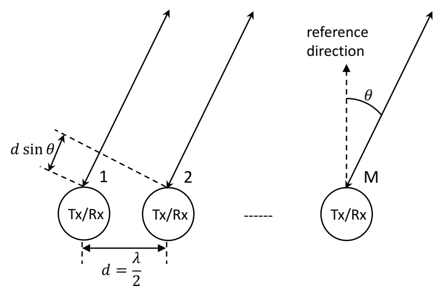

Fig. 1. MIMO transceiver with M antennas and θ is the spacial direction of interest.

$\left[\mathbf{x}^T(1), \mathbf{x}^T(2), \ldots, \mathbf{x}^T(N)\right]^T$ denote the waveform vector.

The signal at a target location with angle θ ($\theta \in \Theta$, which is the angle set) is represented by

$\sum_{m=1}^{M} e^{-j\pi(m-1)\sin\theta} x_m(n) = \mathbf{a}^T(\theta)\mathbf{x}(n),\ n = 1, \ldots, N,$

where $\mathbf{a}(\theta)$ is the transmit steering vector written as $\mathbf{a}(\theta) = \left[1, e^{-j\pi\sin\theta}, \ldots, e^{-j\pi(M-1)\sin\theta}\right]^T$. Then, the power for the probing signal $\mathbf{x}$ at location θ which is named the *transmit beampattern* can be written as follows:

$$\begin{aligned} P(\theta, \mathbf{x}) &= \textstyle\sum_{n=1}^{N} \left(\mathbf{a}^T(\theta)\mathbf{x}(n)\right)^* \left(\mathbf{a}^T(\theta)\mathbf{x}(n)\right) \\ &= \left(\left(\mathbf{I}_N \otimes \mathbf{a}^T(\theta)\right)\mathbf{x}\right)^H \left(\left(\mathbf{I}_N \otimes \mathbf{a}^T(\theta)\right)\mathbf{x}\right) \\ &= \mathbf{x}^H \left(\mathbf{I}_N \otimes \mathbf{a}^*(\theta)\mathbf{a}^T(\theta)\right)\mathbf{x} = \mathbf{x}^H \mathbf{A}(\theta)\mathbf{x}, \end{aligned}$$

where $\mathbf{A}(\theta) = \mathbf{I}_N \otimes \mathbf{a}^*(\theta)\mathbf{a}^T(\theta)$.

Suppose there are $\overline{K}$ targets of interest, and then the spatial cross-correlation sidelobes (cross-correlation beampattern) between the probing signals at locations θ_i and θ_j ($i \neq j, i, j = 1, \ldots, \overline{K}$ and $\theta_i, \theta_j \in \Theta$) is given by

$$\begin{aligned} P_{cc}(\theta_i, \theta_j, \mathbf{x}) &= \textstyle\sum_{n=1}^{N} \left(\mathbf{a}^T(\theta_i)\mathbf{x}(n)\right)^* \left(\mathbf{a}^T(\theta_j)\mathbf{x}(n)\right) \\ &= \left(\left(\mathbf{I}_N \otimes \mathbf{a}^T(\theta_i)\right)\mathbf{x}\right)^H \left(\left(\mathbf{I}_N \otimes \mathbf{a}^T(\theta_j)\right)\mathbf{x}\right) \\ &= \mathbf{x}^H \left(\mathbf{I}_N \otimes \mathbf{a}^*(\theta_i)\mathbf{a}^T(\theta_j)\right)\mathbf{x} = \mathbf{x}^H \mathbf{A}(\theta_i, \theta_j)\mathbf{x}, \end{aligned}$$

where $\mathbf{A}(\theta_i, \theta_j) = \mathbf{I}_N \otimes \mathbf{a}^*(\theta_i)\mathbf{a}^T(\theta_j)$.

The objective of the transmit beampattern matching problem is as follows: i) to match a desired transmit beampattern denoted as $p(\theta)$, which can be formulated as follows[1]:

$$J(\alpha, \mathbf{x}) = \textstyle\sum_{\theta\in\Theta} \omega(\theta) \left|\alpha p(\theta) - P(\theta, \mathbf{x})\right|^2, \quad (1)$$

where $\omega(\theta) \geq 0$ is the weight for the direction θ; and ii) to minimize the cross-correlation between the probing signals at a number of given target locations due to the fact that the statistical performance of adaptive MIMO radar techniques rely on the cross-correlation beampattern, which is given as

$$E(\mathbf{x}) = \textstyle\sum_{\theta_i, \theta_j \in \overline{\Theta},\, i \neq j} \left|P_{cc}(\theta_i, \theta_j, \mathbf{x})\right|^2. \quad (2)$$

Then, by considering $J(\alpha, \mathbf{x})$ and $E(\mathbf{x})$, the MIMO transmit beampattern matching problem is formulated as follows:

$$\begin{aligned} &\underset{\alpha, \mathbf{x}}{\text{minimize}} && f(\alpha, \mathbf{x}) \triangleq J(\alpha, \mathbf{x}) + \omega_{cc} E(\mathbf{x}) \\ &\text{subject to} && \mathbf{x} \in \mathcal{X} \triangleq \mathcal{X}_0 \cap (\cap_i \mathcal{X}_i), \end{aligned} \quad (3)$$

[1] Variable α is introduced since $p(\theta)$ is typically given in a "normalized form" and we want to approximate a scaled version of $p(\theta)$, not $p(\theta)$ itself.

where ω_{cc} controls the sidelobe term, $\mathcal{X}$ generally denotes the waveform constraint, and $\mathcal{X}_0 = \left\{\mathbf{x} \in \mathbb{C}^{MN} \mid \|\mathbf{x}\|_2^2 = c_e^2\right\}$ representing the **total transmit energy (power) constraint**. We are also interested in other practical waveform constraints. **i) Constant modulus constraint** is to prevent the non-linearity distortion of the power amplifier to maximize the efficiency of the transmitter, which is given by $\mathcal{X}_1 = \left\{\mathbf{x} \mid |x(l)| = c_d = \frac{c_e}{\sqrt{MN}}\right\}$ for $l = 1, \ldots, MN$. **ii) Peak-to-Average Ratio (PAR) constraint** is the ratio of the peak signal power to its average power ($\mathrm{PAR}(\mathbf{x}) = \frac{\max|x(l)|^2}{\|\mathbf{x}\|_2^2/MN}$ with $1 \leq \mathrm{PAR}(\mathbf{x}) \leq MN$). The $\mathrm{PAR}(\mathbf{x})$ is constrained to a small threshold, so that the analog-to-digital and digital-to-analog converters can have lower dynamic range, and fewer linear power amplifiers are needed. Since $\mathcal{X}_0$, the PAR constraint is $\mathcal{X}_2 = \left\{\mathbf{x} \mid |x(l)| \leq c_p, \frac{c_e}{\sqrt{MN}} \leq c_p \leq c_e\right\}$ for $l = 1, \ldots, MN$. **iii) Similarity constraint** is to allow the designed waveforms to lie in the neighborhood of a reference one which already can attain a good performance [20], which is denoted as $\mathcal{X}_3 = \left\{\mathbf{x} \mid |\mathbf{x} - \mathbf{x}_{\mathrm{ref}}| \leq c_\epsilon, 0 \leq c_\epsilon \leq \frac{2}{\sqrt{MN}}\right\}$.

Problem (3) is a constrained nonconvex problem due to the nonconvex objective and constraints. We are trying to solve it by using efficient nonconvex optimization methods.

3. PROBLEM SOLVING VIA THE MM METHOD

3.1. The Majorization-Minimization (MM) Method

The MM method [15, 21, 16] is a generalization of the well-known EM method. For an optimization problem given by

$$\underset{\mathbf{x}}{\text{minimize}} \quad f(\mathbf{x}) \quad \text{subject to} \quad \mathbf{x} \in \mathcal{X},$$

instead of dealing with this problem directly which could be difficult, the MM-based algorithm solves a series of simpler subproblems with surrogate functions that majorize $f(\mathbf{x})$ over $\mathcal{X}$. More specifically, starting from an initial point $\mathbf{x}^{(0)}$, it produces a sequence $\left\{\mathbf{x}^{(k)}\right\}$ by the following update rule:

$$\mathbf{x}^{(k)} \in \arg\min_{\mathbf{x}\in\mathcal{X}} \overline{f}\left(\mathbf{x}, \mathbf{x}^{(k-1)}\right),$$

where the surrogate majorizing function $\overline{f}\left(\mathbf{x}, \mathbf{x}^{(k)}\right)$ satisfies

$$\begin{aligned} \overline{f}\left(\mathbf{x}^{(k)}, \mathbf{x}^{(k)}\right) &= f\left(\mathbf{x}^{(k)}\right), && \forall \mathbf{x}^{(k)} \in \mathcal{X}, \\ \overline{f}\left(\mathbf{x}, \mathbf{x}^{(k)}\right) &\geq f(\mathbf{x}), && \forall \mathbf{x}, \mathbf{x}^{(k)} \in \mathcal{X}, \\ \overline{f}'\left(\mathbf{x}^{(k)}, \mathbf{x}^{(k)}; \mathbf{d}\right) &= f'\left(\mathbf{x}^{(k)}; \mathbf{d}\right), && \forall \mathbf{d}, \text{ s.t. } \mathbf{x}^{(k)} + \mathbf{d} \in \mathcal{X}. \end{aligned}$$

The objective function value is monotonically nonincreasing at each iteration. To use the MM method, the key step is to find a majorizing function to make the subproblem easy to solve, which will be discussed in the following subsections.

3.2. Majorization Steps For The Beampattern Matching Term $J(\alpha, \mathbf{x})$

In this section, we discuss the majorization steps, i.e., how to construct a good majorizing function for the beampattern matching term $J(\alpha, \mathbf{x})$ in (1). First, we have

$$J(\alpha,\mathbf{x})=\sum_{\theta\in\Theta}\omega(\theta)\left|\alpha p(\theta)-P(\theta,\mathbf{x})\right|^2$$
$$=\alpha^2\sum_{\theta\in\Theta}\omega(\theta)p^2(\theta)-2\alpha\sum_{\theta\in\Theta}\omega(\theta)p(\theta)P(\theta,\mathbf{x})$$
$$+\sum_{\theta\in\Theta}\omega(\theta)\left(P(\theta,\mathbf{x})\right)^2,$$

which is a quadratic function in variable α. Then, it follows that the minimum of $J(\alpha,\mathbf{x})$ is attained when

$$\alpha(\mathbf{x})=\sum_{\theta\in\Theta}\omega(\theta)p(\theta)P(\theta,\mathbf{x})/\sum_{\theta\in\Theta}\omega(\theta)p^2(\theta).$$

Substituting $\alpha(\mathbf{x})$ back into $J(\alpha,\mathbf{x})$ and considering $P(\theta,\mathbf{x})=\mathrm{Tr}\left(\mathbf{x}\mathbf{x}^H\mathbf{A}(\theta)\right)=\mathrm{vec}\left(\mathbf{x}\mathbf{x}^H\right)^H\mathrm{vec}\left(\mathbf{A}(\theta)\right)$, we get

$$J(\mathbf{x})=\sum_{\theta\in\Theta}\omega(\theta)\left(\mathrm{vec}\left(\mathbf{x}\mathbf{x}^H\right)^H\mathrm{vec}\left(\mathbf{A}(\theta)\right)\right)^2$$
$$-\left(\sum_{\theta\in\Theta}\omega(\theta)p^2(\theta)\right)^{-1}\left(\mathrm{vec}\left(\mathbf{x}\mathbf{x}^H\right)^H\mathrm{vec}\Big(\sum_{\theta\in\Theta}\omega(\theta)\right.$$
$$\left.\times\,p(\theta)\mathbf{A}(\theta)\Big)\right)^2=\mathrm{vec}\left(\mathbf{x}\mathbf{x}^H\right)^H\mathbf{H}_J\mathrm{vec}\left(\mathbf{x}\mathbf{x}^H\right),$$

where $\mathbf{H}_J=\sum_{\theta\in\Theta}\omega(\theta)\mathrm{vec}\left(\mathbf{A}(\theta)\right)\mathrm{vec}\left(\mathbf{A}(\theta)\right)^H-\Big(\sum_{\theta\in\Theta}\omega(\theta)p^2(\theta)\Big)^{-1}\mathrm{vec}\left(\sum_{\theta\in\Theta}\omega(\theta)p(\theta)\mathbf{A}(\theta)\right)\mathrm{vec}\Big(\sum_{\theta\in\Theta}\omega(\theta)p(\theta)\mathbf{A}(\theta)\Big)^H$, and it is easy to see that $J(\mathbf{x})$ is a quartic function in $\mathbf{x}$. Next, we introduce a useful lemma.

Lemma 1. *Let $\mathbf{A}\in\mathbb{H}^K$ and $\mathbf{B}\in\mathbb{H}^K$ such that $\mathbf{B}\succeq\mathbf{A}$. At any point $\mathbf{x}_0\in\mathbb{C}^K$, the quadratic function $\mathbf{x}^T\mathbf{A}\mathbf{x}$ is majorized by $\mathbf{x}^H\mathbf{B}\mathbf{x}+2\mathrm{Re}\left(\mathbf{x}^H\left(\mathbf{A}-\mathbf{B}\right)\mathbf{x}_0\right)+\mathbf{x}_0^H\left(\mathbf{B}-\mathbf{A}\right)\mathbf{x}_0$.*

Proof. Notice that $\left(\mathbf{x}-\mathbf{x}_0\right)^H\left(\mathbf{B}-\mathbf{A}\right)\left(\mathbf{x}-\mathbf{x}_0\right)\geq 0$. $\square$

Based on Lemma 1, we can choose $\psi_{J,1}\geq\lambda_{\max}\left(\mathbf{H}_J\right)$, and because $\psi_{J,1}\mathbf{I}\succeq\mathbf{H}_J$, at iterate $\mathbf{x}^{(t)}$ we have

$$J(\mathbf{x})\leq\psi_{J,1}\mathrm{vec}\left(\mathbf{x}\mathbf{x}^H\right)^H\mathrm{vec}\left(\mathbf{x}\mathbf{x}^H\right)$$
$$+2\mathrm{Re}\left(\mathrm{vec}\left(\mathbf{x}\mathbf{x}^H\right)^H\left(\mathbf{H}_J-\psi_{J,1}\mathbf{I}\right)\mathrm{vec}\left(\mathbf{x}^{(t)}\mathbf{x}^{(t)H}\right)\right)$$
$$+\mathrm{vec}\left(\mathbf{x}^{(t)}\mathbf{x}^{(t)H}\right)^H\left(\psi_{J,1}\mathbf{I}-\mathbf{H}_J\right)\mathrm{vec}\left(\mathbf{x}^{(t)}\mathbf{x}^{(t)H}\right),$$

where since $\mathrm{vec}\left(\mathbf{x}\mathbf{x}^H\right)^H\mathrm{vec}\left(\mathbf{x}\mathbf{x}^H\right)=\|\mathbf{x}\|_2^4=c_e^4$, the first term is just a constant. Then after ignoring the constant terms, we get the following majorizing function for $J(\mathbf{x})$:

$$\overline{J}_1\left(\mathbf{x},\mathbf{x}^{(t)}\right)$$
$$\simeq 2\mathrm{Re}\left(\mathrm{vec}\left(\mathbf{x}\mathbf{x}^H\right)^H\left(\mathbf{H}_J-\psi_{J,1}\mathbf{I}\right)\mathrm{vec}\left(\mathbf{x}^{(t)}\mathbf{x}^{(t)H}\right)\right),$$

where "$\simeq$" stands for "equivalence" up to additive constants. Substituting $\mathbf{H}_J$ back into function $\overline{J}_1\left(\mathbf{x},\mathbf{x}^{(t)}\right)$ and dropping the constants, we have

$$\overline{J}_1\left(\mathbf{x},\mathbf{x}^{(t)}\right)\simeq 2\mathbf{x}^H\left(\mathbf{M}_J-\psi_{J,1}\mathbf{x}^{(t)}\mathbf{x}^{(t)H}\right)\mathbf{x},\quad(4)$$

where $\mathbf{M}_J=\sum_{\theta\in\Theta}\omega(\theta)\left(P\left(\theta,\mathbf{x}^{(t)}\right)-p(\theta)\alpha\left(\mathbf{x}^{(t)}\right)\right)\mathbf{A}(\theta)$. It is easy to see that after majorization, the majorizing function $\overline{J}_1\left(\mathbf{x},\mathbf{x}^{(t)}\right)$ becomes quadratic in $\mathbf{x}$ rather than quartic in $J(\mathbf{x})$. However, using this function as the objective to solve is still hard due to the waveform constraint $\mathcal{X}$.[2] So we propose to majorize $\overline{J}_1\left(\mathbf{x},\mathbf{x}^{(t)}\right)$ again to simplify the problem to solve in each iteration. Thus, we can consider choosing $\psi_{J,2}\geq\lambda_{\max}\left(\mathbf{M}_J\right)\geq\lambda_{\max}\left(\mathbf{M}_J-\psi_{J,1}\mathbf{x}^{(t)}\mathbf{x}^{(t)H}\right)$ for majorization, where we can have the following useful property.

[2] It is a NP-hard unimodular quadratic program even only considering $\mathcal{X}_1$.

Lemma 2. *[22, 23] Define* $\mathbf{B}=\sum_{\theta\in\Theta}\omega(\theta)\left(P\left(\theta,\mathbf{x}^{(t)}\right)-p(\theta)\alpha\left(\mathbf{x}^{(t)}\right)\right)\mathbf{a}^*(\theta)\mathbf{a}^T(\theta)=\begin{bmatrix} b_0 & b_1^* & \cdots & b_{M-1}^* \\ b_1 & b_0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & b_1^* \\ b_{M-1} & \dots & b_1 & b_0 \end{bmatrix},$ *which is Hermitian Toeplitz, $\mathbf{F}$ as a $2M\times 2M$ FFT matrix, and $\mathbf{b}=\left[b_0,b_1,\ldots,b_{M-1},0,b_{M-1}^*,\ldots,b_1^*\right]^T$. Then, we have $\mathbf{M}_J=\mathbf{I}_N\otimes\mathbf{B}$, $\lambda_{\max}\left(\mathbf{M}_J\right)=\lambda_{\max}\left(\mathbf{B}\right)$, and $\lambda_{\max}\left(\mathbf{B}\right)\leq\lambda_\mu=\frac{1}{2}\left(\max_{1\leq i\leq M}\mu_{2i}+\max_{1\leq i\leq M}\mu_{2i-1}\right)$, where $\boldsymbol{\mu}=\mathbf{F}\mathbf{b}$, which is the discrete Fourier transform for $\mathbf{b}$.*

Lemma 2 provides an easy way for the computation of $\psi_{J,2}$. Based on Lemma 1 and using $\psi_{J,2}=\lambda_\mu$, the majorizing function $\overline{J}_1\left(\mathbf{x},\mathbf{x}^{(t)}\right)$ can be further majorized as

$$\overline{J}_1\left(\mathbf{x},\mathbf{x}^{(t)}\right)\leq 2\psi_{J,2}\mathbf{x}^H\mathbf{x}+4\mathrm{Re}\Big(\mathbf{x}^H\Big(\mathbf{M}_J-\psi_{J,1}\mathbf{x}^{(t)}\mathbf{x}^{(t)H}$$
$$-\psi_{J,2}\mathbf{I}\Big)\mathbf{x}^{(t)}\Big)+2\mathbf{x}^{(t)H}\left(\psi_{J,2}\mathbf{I}-\mathbf{M}_J+\psi_{J,1}\mathbf{x}^{(t)}\mathbf{x}^{(t)H}\right)\mathbf{x}^{(t)},$$

where since $\|\mathbf{x}\|_2^2=c_e^2$, the first term is a constant. Then by ignoring the constant terms, the objective becomes a linear majorizing function at iterate $\mathbf{x}^{(t)}$ as follows:

$$\overline{J}_2\left(\mathbf{x},\mathbf{x}^{(t)}\right)\simeq -4\mathrm{Re}\left(\mathbf{x}^H\mathbf{y}_J\right),\quad(5)$$

where $\mathbf{y}_J=-\left(\mathbf{M}_J-c_e^2\psi_{J,1}\mathbf{I}-\psi_{J,2}\mathbf{I}\right)\mathbf{x}^{(t)}$.

3.3. Majorization Steps For The Sidelobe Term $E(\mathbf{x})$

To deal with the sidelobe term $E(\mathbf{x})$ in (2), the majorization steps are similar to $J(\mathbf{x})$. First, we have

$$E(\mathbf{x})=\sum_{\theta_i,\theta_j\in\overline{\Theta},\,i\neq j}\left|P_{cc}\left(\theta_i,\theta_j,\mathbf{x}\right)\right|^2$$
$$=\mathrm{vec}\left(\mathbf{x}\mathbf{x}^H\right)^H\mathbf{H}_E\mathrm{vec}\left(\mathbf{x}\mathbf{x}^H\right),$$

where $\mathbf{H}_E=\sum_{\theta_i,\theta_j\in\overline{\Theta},\,i\neq j}\mathrm{vec}\left(\mathbf{A}\left(\theta_i,\theta_j\right)\right)\mathrm{vec}\left(\mathbf{A}\left(\theta_i,\theta_j\right)\right)^H$. Then, based on Lemma 1, by choosing $\psi_{E,1}\geq\lambda_{\max}\left(\mathbf{H}_E\right)$ and $\psi_{E,2}\geq\lambda_{\max}\left(\mathbf{M}_E-\psi_{E,1}\mathbf{x}^{(t)}\mathbf{x}^{(t)H}\right)$, we can get the majorizing functions at iterate $\mathbf{x}^{(t)}$ written as follows:

$$\overline{E}_1\left(\mathbf{x},\mathbf{x}^{(t)}\right)\simeq 2\mathbf{x}^H\left(\mathbf{M}_E-\psi_{E,1}\mathbf{x}^{(t)}\mathbf{x}^{(t)H}\right)\mathbf{x}$$
$$\leq\overline{E}_2\left(\mathbf{x},\mathbf{x}^{(t)}\right)\simeq -4\mathrm{Re}\left(\mathbf{x}^H\mathbf{y}_E\right),\quad(6)$$

where $\mathbf{M}_E=\sum_{\theta_i,\theta_j\in\overline{\Theta},\,i\neq j}P_{cc}\left(\theta_j,\theta_i,\mathbf{x}^{(t)}\right)\mathbf{A}\left(\theta_i,\theta_j\right)$ and $\mathbf{y}_E=-\left(\mathbf{M}_E-c_e^2\psi_{E,1}\mathbf{I}-\psi_{E,2}\mathbf{I}\right)\mathbf{x}^{(t)}$.

3.4. Solving The Majorized Subproblem in MM

By combing the two majorizing functions $\overline{J}_2\left(\mathbf{x},\mathbf{x}^{(t)}\right)$ and $\overline{E}_2\left(\mathbf{x},\mathbf{x}^{(t)}\right)$, the overall majorizing function at iterate $\mathbf{x}^{(k)}$ for the objective $f(\mathbf{x})$ is given as follows:

$$f(\mathbf{x})\leq\overline{f}\left(\mathbf{x},\mathbf{x}^{(t)}\right)=\overline{J}_2\left(\mathbf{x},\mathbf{x}^{(t)}\right)+\omega_{cc}\overline{E}_2\left(\mathbf{x},\mathbf{x}^{(t)}\right)$$
$$\simeq -4\mathrm{Re}\left(\mathbf{x}^H\mathbf{y}_J\right)-4\omega_{cc}\mathrm{Re}\left(\mathbf{x}^H\mathbf{y}_E\right)=-\mathrm{Re}\left(\mathbf{x}^H\mathbf{y}\right),$$

where $\mathbf{y} = -4\left(\mathbf{M}_J + \omega_{cc}\mathbf{M}_E - c_e^2\left(\psi_{J,1} + \omega_{cc}\psi_{E,1}\right)\mathbf{I} - \left(\psi_{J,2} + \omega_{cc}\psi_{E,2}\right)\mathbf{I}\right)\mathbf{x}^{(t)}$.

Finally, by majorizing the objective function in (3) using the MM method, the subproblem we need to solve at each iteration is given as follows:

$$\min_{\mathbf{x}} \quad \overline{f}\left(\mathbf{x}, \mathbf{x}^{(t)}\right) \simeq -\mathrm{Re}\left(\mathbf{x}^H \mathbf{y}\right) \quad \text{s.t.} \quad \mathbf{x} \in \mathcal{X} \ . \quad (7)$$

For problem (7), as to different interested waveform constraints, closed-form optimal solutions $\mathbf{x}^\star$ can be derived, which are summarized in the following lemma.

Lemma 3. ***i)*** *For fixed energy constraint (i.e., $\mathcal{X} = \mathcal{X}_0$), $\mathbf{x}^\star = c_e \mathbf{y}/\left\|\mathbf{y}\right\|_2$;* ***ii)*** *for constant modulus constraint (i.e., $\mathcal{X} = \mathcal{X}_1$), $\mathbf{x}^\star = c_d e^{j\arg(\mathbf{y})}$;*[3] ***iii)*** *for fixed energy with PAR constraint (i.e., $\mathcal{X} = \mathcal{X}_0 \cap \mathcal{X}_2$), the solution $\mathbf{x}^\star$ can be found in [24, Alg. 2];* ***iv)*** *for constant modulus with similarity constraint (i.e., $\mathcal{X} = \mathcal{X}_1 \cap \mathcal{X}_3$), the solution $\mathbf{x}^\star$ can be found in [25].*

3.5. The MM-Based Beampattern Matching Algorithm

Based on the MM method, in order to solve the original problem (3), we just need to iteratively solve the subproblem (7) with a closed-form solution update in Lemma 3 at each iteration. The overall algorithm is summarized as follows.

Input: $\mathbf{a}(\theta)$, $p(\theta)$, $\mathbf{x}^{(0)}$ and $t = 0$.
Repeat
1. Compute $\mathbf{M}_J$, $\mathbf{M}_E$, $\psi_{J,1}$, $\psi_{E,1}$, $\psi_{J,2}$, $\psi_{E,2}$ and $\mathbf{y}$;
2. Update $\mathbf{x}^{(t)}$ in a closed-form according to Lemma 3;
3. $t = t + 1$;

Until $\mathbf{x}$ and $f(\mathbf{x})$ satisfy a termination criterion.
Output: α, $\mathbf{x}$.

4. NUMERICAL SIMULATIONS

The performance of the proposed algorithm for MIMO transmit beampattern matching is evaluated by numerical simulations. A colocated MIMO radar system is considered with a ULA comprising $M = 10$ antennas with half-wavelength spacing between adjacent antennas. Without loss of generality, the total transmit power is set to $c_e^2 = 1$. Each transmit pulse has $N = 32$ samples. The range of angle is $\Theta = (-90°, 90°)$ with spacing $1°$ under which the weight $\omega(\theta) = 1$ for $\theta \in \Theta$, and $\omega_{cc} = 0$, which is the same setting as [13]. We consider a desired beampattern with three targets or mainlobes ($K = 3$) at $\theta_1 = -40°$, $\theta_2 = 0°$, $\theta_3 = 40°$, and each width of them is $\triangle\theta = 20°$. The desired beampattern is

$$p(\theta) = \begin{cases} 1, & \theta \in \left[\theta_k - \triangle\theta/2, \theta_k + \triangle\theta/2\right], \ k = 1, 2, K \\ 0, & \text{otherwise.} \end{cases}$$

We compare the convergence property over iterations of the objective function for the beampattern matching problem under unimodulus waveform constraint by using the proposed MM-based algorithm (denoted as MM-based algorithm (prop.)) and the ADMM-based algorithm in [13] (denoted as ADMM-based algorithm) , which is shown in Fig. 2.

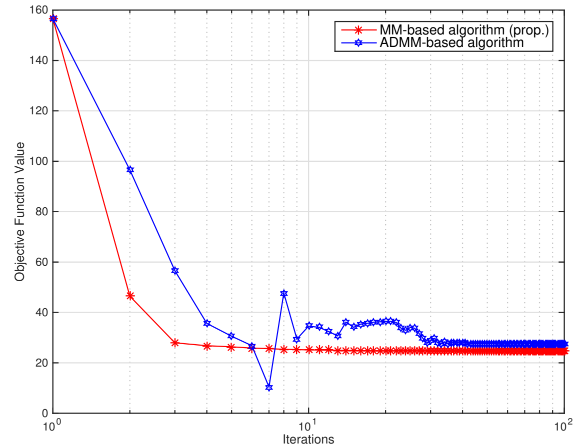

Fig. 2. Convergence comparison for objective function value.

As shown in Fig. 2, the MM-based algorithm can have a monotonic convergence property. And it can converge within 20 iterations which is faster than the benchmark algorithm.

Then, we also compare the matching performance of the designed beampatterns in terms of the mean-squared error (MSE) defined as

$$\mathrm{MSE}\left(P\left(\theta, \mathbf{x}\right)\right) = \mathbb{E}\left[\sum_{\theta \in \Theta} \omega\left(\theta\right)\left|\alpha p\left(\theta\right) - P\left(\theta, \mathbf{x}\right)\right|^2\right].$$

In Fig. 3, we show the simulation results for $\mathrm{MSE}\left(P\left(\theta, \mathbf{x}\right)\right)$ by using different design methods.

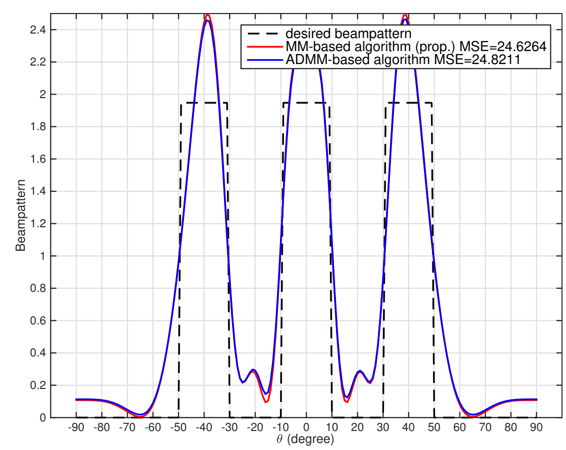

Fig. 3. Transmit beampattern design with 3 targets .

From Fig. 3, we can see that compared to the benchmark, our proposed algorithm can have a tighter matching performance and can obtain a lower MSE. Based on these, the proposed algorithm is validated.

5. CONCLUSIONS

This paper has considered the MIMO transmit beampattern matching problem. Efficient algorithms have been proposed based on the MM method. Numerical simulations show that the proposed algorithms are efficient in solving the beampattern matching problem and can obtain a better performance compared to the the state-of-art method.

[3]The operation $\arg(\mathbf{y})$ is applied element-wise for $\mathbf{y}$.

6. REFERENCES

[1] J. Li and P. Stoica, *MIMO Radar Signal Processing*. Wiley, 2008.

[2] D. Bliss and K. Forsythe, "Multiple-input multiple-output (MIMO) radar and imaging: degrees of freedom and resolution," in *Proc. of the 37th Asilomar Conf. on Signals, Systems and Computers.*, vol. 1. IEEE, 2003, pp. 54–59.

[3] E. Fishler, A. Haimovich, R. Blum, D. Chizhik, L. Cimini, and R. Valenzuela, "MIMO radar: An idea whose time has come," in *Proc. of the 2004 IEEE Radar Conf.*. IEEE, 2004, pp. 71–78.

[4] K. Forsythe, D. Bliss, and G. Fawcett, "Multiple-input multiple-output MIMO radar: Performance issues," in *Proc. of the 38th Asilomar Conf. on Signals, Systems and Computers.*, vol. 1. IEEE, 2004, pp. 310–315.

[5] M. Skolnik, "Radar handbook," *New York: McGraw-Hill*, 1990.

[6] D. R. Fuhrmann and G. San Antonio, "Transmit beamforming for MIMO radar systems using partial signal correlation," in *Proc. of the 38th Asilomar Conf. on Signals, Systems and Computers.*, vol. 1. IEEE, 2004, pp. 295–299.

[7] ——, "Transmit beamforming for MIMO radar systems using signal cross-correlation," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 44, no. 1, 2008.

[8] P. Stoica, J. Li, and Y. Xie, "On probing signal design for MIMO radar," *IEEE Trans. Signal Process.*, vol. 55, no. 8, pp. 4151–4161, 2007.

[9] J. Lipor, S. Ahmed, and M.-S. Alouini, "Fourier-based transmit beampattern design using MIMO radar," *IEEE Trans. Signal Process.*, vol. 62, no. 9, pp. 2226–2235, 2014.

[10] T. Bouchoucha, S. Ahmed, T. Al-Naffouri, and M.-S. Alouini, "DFT-based closed-form covariance matrix and direct waveforms design for MIMO radar to achieve desired beampatterns," *IEEE Trans. Signal Process.*, vol. 65, no. 8, pp. 2104–2113, 2017.

[11] P. Stoica, J. Li, and X. Zhu, "Waveform synthesis for diversity-based transmit beampattern design," *IEEE Trans. Signal Process.*, vol. 56, no. 6, pp. 2593–2598, 2008.

[12] Y.-C. Wang, X. Wang, H. Liu, and Z.-Q. Luo, "On the design of constant modulus probing signals for MIMO radar," *IEEE Trans. Signal Process.*, vol. 60, no. 8, pp. 4432–4438, 2012.

[13] Z. Cheng, Z. He, S. Zhang, and J. Li, "Constant modulus waveform design for MIMO radar transmit beampattern," *IEEE Trans. Signal Process.*, vol. 65, no. 18, pp. 4912–4923, 2017.

[14] S. Boyd, N. Parikh, E. Chu, B. Peleato, and J. Eckstein, "Distributed optimization and statistical learning via the alternating direction method of multipliers," *Found. Trends Mach. Learn.*, vol. 3, no. 1, pp. 1–122, 2011.

[15] D. R. Hunter and K. Lange, "A tutorial on MM algorithms," *Amer. Statist.*, vol. 58, no. 1, pp. 30–37, 2004.

[16] Y. Sun, P. Babu, and D. P. Palomar, "Majorization-minimization algorithms in signal processing, communications, and machine learning," *IEEE Trans. Signal Process.*, vol. 65, no. 3, pp. 794–816, 2016.

[17] J. Song, P. Babu, and D. P. Palomar, "Optimization methods for designing sequences with low autocorrelation sidelobes," *IEEE Trans. Signal Process.*, vol. 63, no. 15, pp. 3998–4009, 2015.

[18] Z. Zhao and D. P. Palomar, "Mean-reverting portfolio with budget constraint," *IEEE Trans. on Signal Process.*, vol. PP, no. 99, p. 1, 2018.

[19] J. Li and P. Stoica, "MIMO radar with colocated antennas," *IEEE Signal Process. Mag.*, vol. 24, no. 5, pp. 106–114, 2007.

[20] J. Li, J. R. Guerci, and L. Xu, "Signal waveform's optimal-under-restriction design for active sensing," *IEEE Signal Process. Lett.*, vol. 13, no. 9, pp. 565–568, Sep. 2006.

[21] M. Razaviyayn, M. Hong, and Z.-Q. Luo, "A unified convergence analysis of block successive minimization methods for nonsmooth optimization," *SIAM J. Optim.*, vol. 23, no. 2, pp. 1126–1153, 2013.

[22] P. Jorge and S. Ferreira, "Localization of the eigenvalues of Toeplitz matrices using additive decomposition, embedding in circulants, and the fourier transform," in *Proc. Symp. Syst. Identif.*, vol. 3, 1994, pp. 271–275.

[23] R. A. Horn and C. R. Johnson, *Matrix analysis*. Cambridge Univ. Press, 1990.

[24] J. A. Tropp, I. S. Dhillon, R. W. Heath, and T. Strohmer, "Designing structured tight frames via an alternating projection method," *IEEE Trans. Inf. Theory*, vol. 51, no. 1, pp. 188–209, 2005.

[25] L. Zhao and D. P. Palomar, "Maximin joint optimization of transmitting code and receiving filter in radar and communications," *IEEE Trans. Signal Process.*, vol. 65, no. 4, pp. 850–863, 2017.